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Energy and CO_2 emission performance in electricity generation: A non-radial directional distance function approach

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ABSTRACT

This paper presents a non-radial directional distance function approach to modeling energy and CO_2emission performance in electricity generation from the production efficiency point of view. We first define and construct the environmental production technologies for the countries with and without CHP plants, respectively. The non-radial direction distance function approach is then proposed and several indexes are developed to measure energy and CO_2 emission performance of electricity generation. The directional distance functions established can be computed by solving a series of data envelopment analysis models. We then conduct an empirical study using the dataset for over one hundred countries. It is found that OECD countries have better carbon emission performance and integrated energy-carbon performance than non-OECD countries in electricity generation, while the difference in energy performance is not significant.

1. Introduction

Electricity generation contributes to over a third of the global energy-related CO_2 emissions (Ang et al., 2011). It is therefore worthwhile to benchmark the energy performance of electricity generation and assess its potential for CO_2 emission reduction. Several studies, such as Graus and Worrell (2009) and Maruyama and Eckelman (2009), analyze the emissions reduction potential in electricity generation for various countries based on the assumption that the efficiencies of fossil-fuel electricity generation were to improve to certain levels. Ang et al. (2011) estimates the potential for reducing CO_2 emissions arising from electricity generation in over 100 countries through improving generation efficiency and increasing the share of non-fossil fuel generation.

In these studies, it is assumed that the electricity generation efficiency for each fossil-fuel type in a country will reach a certain percentile level calculated based on the world/regional generation efficiencies. In practice, improving electricity generation efficiency nationwide is not straightforward. It requires substantial efforts in technology innovation and financial investments. In addition, these previous studies often consider one indicator at a time while it is clearly more meaningful to consider several indicators simultaneously in the benchmark analysis. Furthermore, for countries that have combined heat and power (CHP) plants, the heat energy produced from CHP plants has to be first converted to its electricity equivalent for estimating generation efficiency and this may bring uncertainty into the benchmark analysis.[1]

Different from the benchmarking studies introduced above, this paper attempts to model energy and CO_2 emission performance in electricity generation from the production efficiency point of view. The relevant indicators will be modelled within a joint production framework of desirable and undesirable outputs, and both energy and CO_2 emission performance can be analyzed simultaneously. In addition, heat energy generated from CHP plants can be treated as a separate desirable output, which avoids the need to convert heat energy to its electricity equivalent.

In literature, the production efficiency approach and especially data envelopment analysis (DEA),[2] has been widely employed to model energy or environmental performance (Zhou et al., 2008). Examples of such studies include Zofio and Prieto (2001), Zhou et al. (2007, 2010a), Kortelainen (2008), Camarero et al. (2008), Lozano and Gutierrez (2008), Picazo-Tadeo and Prior (2009), Mukherjee (2010), Sueyoshi and Goto (2011a,b,c, 2012) and Picazo-Tadeo et al. (in press). With regards to the electricity sector, the survey by Zhou et al. (2008) provides a number of examples in which DEA has been employed to assess the relative efficiency of electricity

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[1] For the complications in electricity generation efficiency computation that arise from CHP and the approach to resolving them, see Ang et al. (2011) and IEA (2008).
[2] For an overview of main methodological developments in DEA over the past three decades, see Cook and Seiford (2009).

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generation utilities. Recently, Welch and Barnum (2009) use DEA to analyze the environmental and economic tradeoffs of different fossil-fuel power plants. Yang and Pollitt (2009) assess the performance of Chinese coal-fired power plants by incorporating both undesirable outputs and uncontrollable variables. Sueyoshi et al. (2010) and Sueyoshi and Goto (2011a,b,c) develop several novel DEA models for assessing the unified performance of operational and environmental efficiencies in fossil-fuel electricity generation.

Previous studies show that performance measurement of electricity generation sector by separating undesirable outputs from desirable outputs would provide additional insights. For instance, the study by Sueyoshi and Goto (2011a) demonstrates that DEA with output separation can provide an encompassing unified efficient measure for fossil-fuel electricity generation. Through classifying outputs into desirable and undesirable ones, Sueyoshi and Goto (2011c) propose new DEA models for measuring not only the returns to scale (for desirable outputs) but also the damages to scale (for undesirable outputs) in electricity generation. Similarly, this paper also contributes to the modeling of the performance of electricity generation within a joint-production framework of desirable and undesirable outputs. Nevertheless, this study differs from the previous studies in the following aspects. First, it proposes the use of non-radial directional distance function approach to modeling energy and CO_2 emission performance in electricity generation. Although there are some previous theoretical contributions related to non-radial directional distance functions, e.g. Fukuyama and Weber (2009, 2010), Färe and Grosskopf (2010), Mahlberg and Sahoo (2011), and Barros et al. (2012), these studies usually directly provide the non-radial DEA models for calculating the directional slacks-based inefficiency measures without formally defining the function itself. Different from these earlier studies, we start from defining the non-radial directional distance function holding some desirable mathematical properties, which is more consistent with the practice of axiomatic approach on efficiency measurement as followed by the directional distance functions. Second, based on the non-radial directional distance function with various directional vectors, we further define several standardized indexes for measuring energy performance, carbon performance and energy-carbon performance of electricity generation separately. This is more consistent with the composite indicator approach to assessing energy efficiency or environmental performance that has been widely adopted in the literature of energy economics and policy (Zhou et al., 2010b). Third, while previous relevant studies mainly focus on the performance measurement of electricity generation at plant/company level, this study assesses the energy and CO_2 emission performance of electricity generation at the economy level with non-radial directional distance function and DEA models.

Technically, we first divide all the countries into two groups, one without and the other with CHP plants.[3] For the first group, energy use, electricity and CO_2 emissions are modeled in a joint-production framework of desirable and undesirable outputs. For the second group, heat generated from CHP plants is treated as another desirable output. Based on the environmental production technologies specified, we propose two non-radial directional distance functions and several energy and CO_2 emission performance indexes, and apply them to the dataset used in Ang et al. (2011). The directional distance functions as well as the energy and CO_2 emission performance indexes are derived through solving several DEA type models.

The rest of this paper is organized as follows. In Section 2, we first introduce the environmental production technologies for countries with and without CHP plants. We then propose the non-radial directional distance functions and develop energy and CO_2 emission performance indexes. The DEA models for solving the non-radial directional distance functions are also proposed. Section 3 presents an empirical study using the proposed approach to modeling the energy and CO_2 emission performance in world electricity generation. Section 4 concludes this study.

[3] Distinguishing between the two groups will make the energy and CO_2 emission performance of electricity generation within each group more comparable as the production frontier constructed and efficiency scores derived are based on the data from homogenous countries. In addition, it helps to avoid the need to convert the heat generated from CHP plants to its electricity equivalent.

2. Methodology

2.1. Environmental production technology

We first model electricity generation within a joint production framework of desirable and undesirable outputs. For the group of countries without CHP plants, assume that F, E and C are respectively fossil fuel consumption (input) in electricity generation, the electricity generated from fossil-fuel power plants (desirable output) and the total CO_2 emissions from these plants (undesirable output).[4] The multiple-output production technology can be described as

$$T_1 = \{(F, E, C) : F \text{ can produce } (E, C)\} \tag{1}$$

In production economics, T_1 may be considered as a generalization to the single-output production technology characterized by production function. It is often assumed that T_1 satisfies the standard axioms, e.g. inactivity is always possible and finite amounts of inputs can only produce finite amounts of outputs. Alternatively, T_1 can also be represented by its equivalent output set $P_1(F)$ such that

$$P_1(F) = \{(E, C) : F \text{ can produce } (E, C)\} \tag{2}$$

In Eq. (1), F and E are often assumed to be strongly or freely disposable. It implies that $(F',E,C) \in T_1$ (or $(F,E',C) \in T_1$) if $(F,E,C) \in T_1$ and $F' \geqslant F$ (or $E' \leqslant E$). If the production technology is represented by output set $P_1(F)$, the strong disposability implies that $(E,C) \in P_1(F')$ (or $(E',C) \in P_1(F)$) if $(E,C) \in P_1(F)$ and $F' \geqslant F$ (or $E' \leqslant E$). More details on multi-output production technology can be found in Färe and Grosskopf (2005).

In order to reasonably model the production technology that produces both desirable and undesirable outputs, as described in Färe et al. (1989), needs to satisfy the assumptions of weak disposability and null-jointness. Technically, the two assumptions can be respectively expressed as.

($\mathbf{a}_1$) If $(F,E,C) \in T_1$ and $0 \leqslant \theta \leqslant 1$, then $(F, \theta E, \theta C) \in T_1$.
($\mathbf{b}_1$) If $(F,E,C) \in T_1$ and $C = 0$, then $E = 0$.

The weak disposability assumption, i.e. ($\mathbf{a}_1$), implies that the reduction of CO_2 emissions in electricity generation is not free but a proportional reduction in both electricity output and CO_2 emissions is feasible. The null-jointness assumption, i.e. ($\mathbf{b}_1$), states that CO_2 emissions are unavoidable in fossil-fuel electricity generation. The only way to eliminate all the CO_2 emissions in fossil-fuel electricity generation is to cease the generation process. Once the two assumptions are imposed, T_1 could be referred to as an environmental production technology.

[4] It should be pointed out that the production technology specified in some earlier studies on examining the performance of electricity generation at the plant level may include non-energy inputs. For instance, Sueyoshi and Goto (2011a) use two non-energy inputs such as generation capacity and number of employees in addition to energy input, electricity output and CO_2 emissions. As our purpose is to assess the energy and CO_2 emission performance of electricity generation at the economy level, only energy input is considered in this study which is similar to the earlier study by Zhou and Ang (2008). Nevertheless, the approach proposed in this paper can be easily adapted to the case, where non-energy inputs are included.

For the group of countries with CHP plants, we further assume that H denotes the heat generated from CHP plants. Similar to electricity, useful heat is also a desirable output. For convenience, fossil fuel consumption, the electricity generated and the CO_2 emissions are still represented by F, E and C, respectively. Then the multiple-output production technology can be represented by

$$T_2 = \{(F, E, H, C) : F \text{ can produce } (E, H, C)\} \tag{3}$$

For T_2, the standard axioms and the strong disposability of input and desirable outputs still hold. T_2 can also be referred to as an environmental production technology if the weak disposability and null-jointness assumptions of desirable and undesirable outputs are imposed. Mathematically, the two assumptions can be described as

($\mathbf{a}_2$) If $(F,E,H,C) \in T_2$ and $0 \leqslant \theta \leqslant 1$, then $(F,\theta E,\theta H,\theta C) \in T_2$.
($\mathbf{b}_2$) If $(F,E,H,C) \in T_2$ and $C = 0$, then $E = H = 0$.

So far we have defined the environmental production technologies T_1 and T_2, which are respectively used to characterize the fossil-fuel electricity generation in countries without and with CHP plants. Since T_1 and T_2 are only conceptually defined and have no concrete forms, they cannot be directly employed in empirical studies. In literature, a common practice is to formulate them within a nonparametric piecewise linear framework. Suppose that there are N countries without CHP plants and M countries with CHP plants. For country n of the N countries without CHP plants, we assume that F_{1n}, E_{1n}, C_{1n} is the vector of its fossil fuel input, electricity output and CO_2 emissions. Then T_1 exhibiting constant returns to scale can be represented by[5]

$$\begin{aligned} T_1 = \{(F, E, C) : & \sum_{n=1}^{N} z_{1n} F_{1n} \leqslant F \\ & \sum_{n=1}^{N} z_{1n} E_{1n} \geqslant E \\ & \sum_{n=1}^{N} z_{1n} C_{1n} = C \\ & z_{1n} \geqslant 0, \quad n = 1, 2, \ldots, N\} \end{aligned} \tag{4}$$

For country m of the M countries with CHP plants, we assume that $(F_{2m}, E_{2m}, H_{2m}, C_{2m})$ is the vector of its fossil fuel input, electricity output, the heat generated from CHP plants and CO_2 emissions. Then T_2 exhibiting constant returns to scale can be represented by

$$\begin{aligned} T_2 = \{(F, E, H, C) : & \sum_{m=1}^{M} z_{2m} F_{2m} \leqslant F \\ & \sum_{m=1}^{M} z_{2m} E_{2m} \geqslant E \\ & \sum_{m=1}^{M} z_{2m} H_{2m} \geqslant H \\ & \sum_{m=1}^{M} z_{2m} C_{2m} = C \\ & z_{2m} \geqslant 0, \quad m = 1, 2, \ldots, M\} \end{aligned} \tag{5}$$

It can be easily verified that both T_1 and T_2 satisfy the weak disposability and null-jointness assumptions of desirable and undesirable outputs as discussed earlier. As Eqs. (5) and (6) are consistent with the production possibility set characterized by DEA models, they can be referred to as environmental DEA technologies (Färe and Grosskopf, 2004). In empirical studies, environmental DEA technologies exhibiting constant returns to scale have been widely applied, such as those listed in Zhou et al. (2008a).

2.2. Non-radial directional distance functions

A large number of studies, such as Zofio and Prieto (2001), Zhou et al. (2007, 2008b, 2010), Kortelainen (2008), Kuosmanen and Kuosmanen (2009), and Sueyoshi and Goto (2011a,b,c, 2012), have contributed to the development of DEA models for measuring energy efficiency or environmental performance. Many earlier studies define energy efficiency or environmental performance indexes based on the Shephard distance functions and then use the DEA technique to compute the index values. However, a new development in this area is to employ the more general directional distance function approach that was originally developed by Chambers et al. (1996, 1998). A major advantage of directional distance function is that it is capable of expanding desirable outputs and contracting inputs/undesirable outputs simultaneously. The Shephard distance function can be considered as a special case of the directional distance function when the direction vector is defined in an appropriate way. Since fossil energy use will inevitably produce undesirable outputs such as CO_2 emissions, the directional distance function approach seems to be an appropriate tool for modeling energy and environmental performance. See, for example, Picazo-Tadeo et al. (2005, 2009), Chang and Hu (2010) and Macpherson et al. (2010). We shall follow this development and propose a non-radial directional distance function to model energy and CO_2 emission performance in electricity generation in this study.

The directional distance function developed by Chambers et al. (1996, 1998) assumes that inputs and undesirable outputs are contracted and desirable outputs are expanded at the same rate, so it may still be treated as a radial measure of efficiency (or inefficiency). From the perspective of axiomatic approach on efficiency measurement, radial measure may be more favourable as the efficiency function has some desirable mathematical characteristics (Sahoo et al., 2011). However, radial efficiency measures may overestimate the efficiency when there exist non-zero slacks (Fukuyama and Weber, 2009).[6] In the context of directional distance function, several studies including Fukuyama and Weber (2009, 2010), Färe and Grosskopf (2010), Mahlberg and Sahoo (2011), Fukuyama et al. (2011) and Barros et al. (2012) investigate how to incorporate the slacks to give a meaningful efficiency/inefficiency measure. Built upon the earlier work by Fukuyama et al. (2011) and Barros et al. (2012), we define the following non-radial directional distance function for the group of countries without CHP plants:

$$\vec{D}_1(F, E, C; g_1) = \sup\left\{\mathbf{w}_1^T \boldsymbol{\beta}_1 : ((F, E, C) + g_1 \cdot diag(\boldsymbol{\beta}_1)) \in T_1\right\} \tag{6}$$

where $\mathbf{w}_1 = (w_{1F}, w_{1E}, w_{1C})^T$ denotes the normalized weight vector that is relevant to the numbers of inputs and outputs,

[5] Kuosmanen (2005) proposes a more general formulation of environmental production technology exhibiting variable returns to scale, which should be treated as an important advancement in modeling weakly disposable production technology. Zhou et al. (2008b) provide more discussions on different environmental DEA technologies in the context of environmental performance measurement. Recently, Podinovski and Kuosmanen (2011) extend Kuosmanen (2005) by relaxing the convexity assumption of output sets. Kuosmanen and Matin (2011) further develop the dual formulation of Kuosmanen technology that facilitates the economic interpretation of weak disposability.

[6] For a discussion on the choice between radial and non-radial efficiency measures, please refer to Sahoo and Tone (2009) and Sahoo et al. (2011).

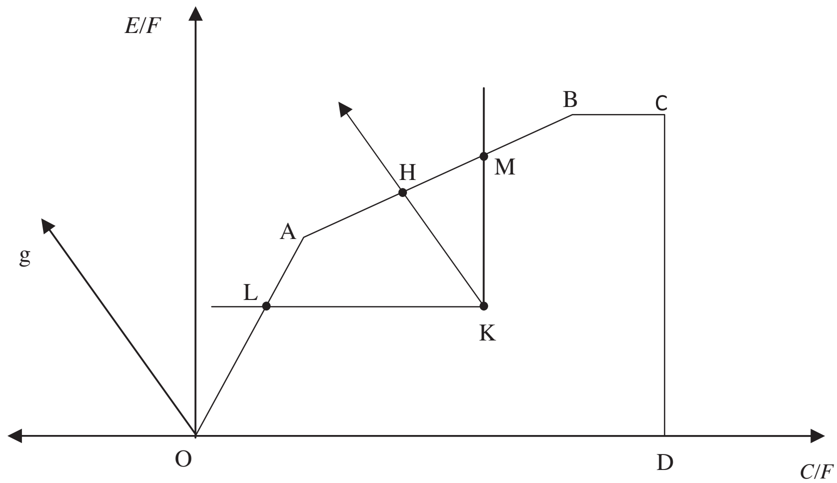

Fig. 1. A graphical illustration of radial and non-radial directional distance functions.

$g_1 = (g_{1F}, g_{1E}, g_{1C})$ is the explicit directional vector in which the input–output combination will be scaled, and $\boldsymbol{\beta}_1 = (\beta_{1F}, \beta_{1E}, \beta_{1C})^T \geqslant 0$ denotes the vector of the scaling factors.[7]

Clearly, the non-radial directional distance function defined in Eq. (6) allows the inputs and outputs to be adjusted non-proportionally. It can be easily shown that the non-radial directional distance function defined in Eq. (6) also satisfies the important properties of directional distance function described in Färe and Grosskopf (2005). Appendix A provides a simple description and proof of several important properties. Mathematically, the value of $\overrightarrow{D}_1(F,E,C;g_1)$ can be obtained by solving the following DEA type model:

$$\begin{aligned}
\overrightarrow{D}_1(F,E,C;g_1) = &\max w_{1F}\beta_{1F} + w_{1E}\beta_{1E} + w_{1C}\beta_{1C}\\
&\text{s.t.} \sum_{n=1}^{N} z_{1n}F_{1n} \leqslant F + \beta_{1F}g_{1F}\\
&\sum_{n=1}^{N} z_{1n}E_{1n} \geqslant E + \beta_{1E}g_{1E}\\
&\sum_{n=1}^{N} z_{1n}C_{1n} = C + \beta_{1C}g_{1C}\\
&z_{1n} \geqslant 0, \quad n = 1,2,\ldots,N, \ \beta_{1F}, \ \beta_{1E}, \ \beta_{1C} \geqslant 0
\end{aligned} \tag{7}$$

Model (7) is externally very similar to the weighted Russell direction distance model as described in Barros et al. (2012). Assume that s_{1F}, s_{1E}, s_{1C} are nonnegative slack variables associated with fuel input, electricity generated and CO_2 emissions. If β_{1F}, β_{1E} and β_{1C} are respectively set equal to $-s_{1F}/g_{1F}$, s_{1E}/g_{1E} and $-s_{1C}/g_{1C}$, then $\overrightarrow{D}_1(F,E,C;g_1)$ would be a weighted version of the slacks-based inefficiency measure defined in Fukuyama and Weber (2010) and Fukuyama et al. (2011). Obviously, $\overrightarrow{D}_1(F,E,C;g_1)$ is affected by the direction vector g_1. If $\overrightarrow{D}_1(F,E,C;g_1) = 0$, it means that the country evaluated is located at the frontier of best practice and is therefore efficient in the g_1 direction. Otherwise, the country being evaluated will be inefficient in the g_1 direction. If the direction vector is set equal to $(-1,1,-1)$, Eq. (7) without undesirable output is almost the same as the generalized directional distance function proposed in Färe and Grosskopf (2010) except that the latter does not consider the numbers of inputs in the objective function.[8] If the direction vector is set equal to $(-F,0,0)$, Eq. (7) is essentially equivalent to an input-oriented DEA model with undesirable outputs.

Fig. 1 provides a simple graphical illustration of the non-radial directional distance function as defined in Eq. (7). In the figure, the area OABCD is assumed to be the output set (deflated by the fossil input) corresponding to the environmental DEA technology as defined in Eq. (4). For point K, if the direction g is taken and the traditional directional distance function is used, H would be the benchmark point for evaluating K. However, if the non-radial directional distance function is used, the benchmarking point would be located at any point of the polygonal line LAM. On the other hand, if the Shephard input distance function for CO_2 emissions or the Shephard output distance function for electricity output is used, the benchmark point would be L or M. It implies that the non-radial directional distance function is more general and flexible than the traditional directional distance function or Shephard distance function in efficiency measurement.

We shall now employ Eq. (7) to model the energy and CO_2 emission performance of electricity generation in the countries without CHP plants by setting various directional vectors. As the first possibility, we set g_1 equal to $(-F,E,0)$. In the circumstance, there are only two scaling factors so that the normalized weight vector is set as $(1/2,1/2,0)$. The resulting DEA type model seeks to reduce fossil energy use and expand electricity output non-proportionally in order to find a benchmark for performance evaluation. Once Eq. (7) is solved, we can easily obtain the potential fossil fuel input and potential electricity output for the country being evaluated. The ratio of actual energy efficiency to potential energy efficiency can then be used to define an energy performance index (EPI_1). Suppose that β^*_{1F} and β^*_{1E} are the optimal solution to Eq. (7) with $(-F,E,0)$ as the direction. The EPI_1 can be formulated as

$$EPI_1 = \frac{E/F}{(E+\beta^*_{1E}E)/(F-\beta^*_{1F}F)} = \frac{1-\beta^*_{1F}}{1+\beta^*_{1E}} \tag{8}$$

[7] It should be pointed out that Färe and Grosskopf (2010) first proposed the basic ideas of non-radial directional distance function without considering undesirable outputs, which was built upon the directional distance function developed by Chambers et al. (1996, 1998). Fukuyama et al. (2011) and Barros et al. (2012) define the directional slacks-based inefficiency measures by incorporating undesirable outputs. A common feature of these earlier studies is that they directly provide the DEA models for calculating the inefficiency measures. Differing from these earlier studies, we first formally define the non-radial directional distance function holding some desirable mathematical properties, based on which the DEA type model is then formulated. As such, our approach is more natural from the point of view of axiomatic production theory.

[8] The generalized directional distance function proposed in Färe and Grosskopf (2010), which is externally similar to the additive DEA model (Cooper et al., 2006), has recently been used by Chang and Hu (2010) to measure total-factor energy productivity.

Obviously, EPI_1 lies between zero and unity. A larger EPI_1 implies better energy performance. If EPI_1 is equal to unity, it means that the country has the best energy performance in electricity generation.

As the second possibility, we set g_1 equal to $(0, E, -C)$ and the normalized weight vector is set as $(0, 1/2, 1/2)$. For simplicity, we assume that β_{1E}^* and β_{1C}^* are the optimal solutions to Eq. (7) with $(0, E, -C)$ being the direction. Then we can define the carbon performance index (CPI) as the ratio of potential carbon intensity to actual carbon intensity. Mathematically, CPI can be expressed as

$$CPI_1 = \frac{(C - \beta_{1C}^* C)/(E + \beta_{1E}^* E)}{C/E} = \frac{1 - \beta_{1C}^*}{1 + \beta_{1E}^*} \tag{9}$$

Similar to EPI_1, CPI_1 also lies between zero and unity. A larger CPI_1 represents better CO_2 emission performance. If CPI_1 is equal to unity, it means that the country has the best CO_2 emission performance in electricity generation.

In order to simultaneously model energy and CO_2 emission performance in electricity generation, we set g_1 equal to $(-F, E, -C)$ and the normalized weight vector as $(1/3, 1/3, 1/3)$. The resulting model is externally similar to the additive DEA model in the sense that both attempt to identify the potential slacks in inputs and outputs as much as possible. For convenience, we still take β_{1E}^*, β_{1F}^* and β_{1C}^* as the optimal solutions to Eq. (7) with $(-F, E, -C)$ as the direction vector. If $\beta_{1F}^* + \beta_{1E}^* + \beta_{1C}^* = 1$, Eq. (7) will have multiple optimal solutions (see Appendix B.1 for a proof).[9] Following the sprit of Eqs. (8) and (9), we can define an energy-carbon performance index (ECPI) as

$$ECPI_1 = \frac{1/2\left((1 - \beta_{1F}^*) + (1 - \beta_{1C}^*)\right)}{1 + \beta_{1E}^*} = \frac{1 - 1/2(\beta_{1F}^* + \beta_{1C}^*)}{1 + \beta_{1E}^*} \tag{10}$$

In Eq. (1), the numerator represents the average proportion by which the fossil fuel input and CO_2 emissions can be reduced, while the denominator measures the degree to which the electricity output can be increased. It should be pointed out that Eq. (10) is very similar to the slacks-based efficiency measure proposed by Tone (2001).[10] As such, it inherits the desirable properties of slacks-based efficiency measures such as unit invariance and monotonicity. Furthermore, $ECPI_1$ is also a standardized index between zero and unity. If $ECPI_1$ is equal to unity, it means that the country is located at the frontier of best practice.

The foregoing discussions are based on the environmental DEA technology . In order to model the energy and CO_2 emission performance of countries with CHP plants, we define the following non-radial directional distance function based on T_2:

$$\vec{D}_2(F, E, H, C; g_2) = \sup\{\mathbf{w}_2^T \boldsymbol{\beta}_2 : (F, E, H, C) + g_2 \cdot diag(\boldsymbol{\beta}_2) \in T_2\} \tag{11}$$

where $\mathbf{w}_2 = (w_{2F}, w_{2E}, w_{2H}, w_{2C})^T$ denotes the normalized weight vector determined by the numbers of inputs and outputs, $g_2 = (g_{2F}, g_{2E}, g_{2H}, g_{2C})$ is the explicit directional vector in which the input–output combination will be scaled, and $\boldsymbol{\beta}_2 = (\beta_{2F}, \beta_{2E}, \beta_{2H}, \beta_{2C}) > 0$ denotes the vector of scaling factors. Like $\vec{D}_1(F, E, C; g_1)$, $\vec{D}_2(F, E, H, C; g_2)$ also satisfies the several properties of directional distance function described in Appendix A. Mathematically, $\vec{D}_2(F, E, H, C; g_2)$ can be obtained by solving the following DEA type model:

$$\begin{aligned}
\vec{D}_2(F, E, H, C; g_2) = \max\ & w_{2F}\beta_{2F} + w_{2E}\beta_{2E} + w_{2H}\beta_{2H} + w_{2C}\beta_{2C} \\
\text{s.t.}\ & \sum_{m=1}^{M} z_{2m} F_{2m} \leqslant F + \beta_{2F} g_{2F} \\
& \sum_{m=1}^{M} z_{2m} E_{2m} \geqslant E + \beta_{2H} g_{2E} \\
& \sum_{m=1}^{M} z_{2m} H_{2m} \geqslant H + \beta_{2H} g_{2H} \\
& \sum_{m=1}^{M} z_{2m} C_{2m} = C + \beta_{2C} g_{2C} \\
& z_{2m} \geqslant 0, \quad m = 1, 2, \ldots, M \\
& \beta_{2F},\ \beta_{2E},\ \beta_{2H},\ \beta_{2C} \geqslant 0
\end{aligned} \tag{12}$$

Similar to Eq. (7), Eq. (12) based on different direction vectors would also lead to different benchmarks and therefore different performance scores. To define an EPI, we first set$g_2 = (-F, E, H, 0)$. The resulting DEA type model would seek to reduce fossil energy use and expand electricity and heat output non-proportionally while not increasing the amount of CO_2 emissions. Since there are two desirable outputs and one input, the normalized weight vector is set as $(1/2, 1/4, 1/4, 0)$ so that the weight for input is the same as that for two desirable outputs. Suppose that β_{2F}^* β_{2E}^* and β_{2H}^* are the optimal solutions to Eq. (12) with $(-F, E, H, 0)$ as the direction, Similar to Eq. (8), we may define an EPI as follows

$$EPI_2 = \frac{1 - \beta_{2F}^*}{1 + 1/2(\beta_{2E}^* + \beta_{2H}^*)} \tag{13}$$

Compared to EPI_1, EPI_2 considers the adjustments for not only electricity but also heat outputs. In the case that electricity and heat outputs cannot be increased further, EPI_2 is equal to the proportion by which fossil input can be reduced.

By setting the direction vector g_2 as $(0, E, H, -C)$ and taking $(0, 1/4, 1/4, 1/2)$ as the normalized weight vector, we can use Eq. (12) to estimate the degrees to which the electricity and heat outputs are increased and the CO_2 emissions is reduced non-proportionally when fossil input is not increased. If β_{2E}^*, β_{2H}^* and β_{2C}^* are the resulting optimal solutions, we can use them to define the following CPI:

$$CPI_2 = \frac{1 - \beta_{2C}^*}{1 + 1/2(\beta_{2E}^* + \beta_{2H}^*)} \tag{14}$$

As the third case, we set the direction vector g_2 equal to $(-F, E, H, -C)$. By assuming the weights for single input, two desirable outputs and single undesirable output are equal to each other, we take $(1/3, 1/6, 1/6, 1/3)$ as the normalized weight vector. The resulting model can be used to estimate the degrees to which the electricity and heat outputs are increased and the fossil input and CO_2 emissions are reduced non-proportionally. Suppose that β_{2F}^*, β_{2E}^*, β_{2H}^* and β_{2C}^* are the optimal solutions. It can be shown that Eq. (12) has multiple optimal solutions if $\beta_{2F}^* + 1/2\beta_{2E}^* + 1/2\beta_{2H}^* + \beta_{2C}^* = 1$ (see Appendix B.2 for a proof). Similar to Eq. (10), we define the following ECPI for modelling energy-carbon performance of electricity generation for the countries with CHP plants.

$$ECPI_2 = \frac{1 - 1/2(\beta_{2C}^* + \beta_{2F}^*)}{1 + 1/2(\beta_{2E}^* + \beta_{2H}^*)} \tag{15}$$

The indexes derived from Eqs. (13)–(15) are also between zero and unity. In particular, the ECPI defined in Eq. (15) is externally consistent with the slacks-based efficiency measure developed in Tone (2001).

[9] It should be pointed out that the occurrence of multiple optimal solutions in DEA-based efficiency measurement has been well investigated by a number of previous studies such as Sueyoshi and Sekitani (2009).

[10] Although the original slacks-based efficiency measure proposed by Tone (2001) does not contain undesirable outputs, Zhou et al. (2006) and Lozano and Gutierrez (2011) show how slacks-based efficiency measures within a joint production framework of desirable and undesirable outputs can be developed.

Table 1
Summary statistics of the variables used for G1 and G2 countries.

	Group 1 (G1)			Group 2 (G2)			
	F (Mtoe)	*E* (TWh)	*C* (Mt)	*F* (Mtoe)	*E* (TWh)	*H* (PJ)	*C* (Mt)
Mean	17.133	69.440	61.060	32.886	138.176	212.970	111.572
Std. dev.	64.589	245.659	252.794	107.913	469.534	863.273	380.583
Min	0.001	0.005	0.003	0.044	0.070	0.007	0.135
Max	545.505	2044.832	2158.246	694.214	3078.074	5735.313	2481.701

3. Empirical study

3.1. Data

The recent study by Ang et al. (2011) on the potential for reducing CO_2 emissions from electricity generation covers 129 countries and is based on the data for the year 2005. In this study, we use the same dataset but excluding three countries with very little electricity generation from fossil fuels. The remaining 126 countries accounted for 97% of the global electricity generation from fossil fuels in 2005.[11]

The 126 countries are classified into two groups: Group 1 without CHP plants and Group 2 with CHP plants (hereafter referred to as G1 and G2 respectively). G1 consists of 82 countries which are mainly non-OECD countries, while G2 consists of the remaining 44 countries which are mainly OECD countries. Of the two groups, G1 takes up a slightly smaller share (48.4%) of the total electricity generation but a marginally larger share (50.5%) of the CO_2 emissions. As defined in Section 2.1, for G1 countries the single input, desirable output and undesirable output are respectively fossil fuel consumption, electricity generated and CO_2 emissions. In addition to the three indicators, G2 has an additional desirable output, i.e. heat generated from CHP plants. Table 1 shows the descriptive statistics of the indicators for G1 and G2 countries. Their units of measurement are million tonnes of oil equivalent (Mtoe), Terawatt-hours (TWh), million tonnes (Mt) of CO_2 emissions and Petajoules (PJ), respectively.[12]

3.2. Results and discussions

The non-radial directional distance function approach proposed in Section 2 has been applied to model the energy and carbon emission performance of each country in electricity generation using the 2005 data. For G1 countries, we employ Eqs. (7)–(10) to compute the EPI_1, CPI_1 and $ECPI_1$ scores for each country. Also, the EPI_2, CPI_2 and $ECPI_2$ scores for each country in G2 are calculated by solving Eqs. (12)–(15). The estimates of non-radial directional distance function and the three indices for ten selected countries are provided in Appendix C.[13] Also, the countries that form the best practice frontier under various assumptions are summarized in Table 2. For G1 countries, it can be found that Spain and Tunisia frequently appear in the best practice frontiers. This could be explained by the facts that Spain has the highest electricity production per unit of fossil fuel consumption, and Tunisia has the lowest CO_2emissions per unit of electricity production as the majority of its electricity production comes from natural gas. For G2 countries, countries such as Switzerland, Lithuania and Ukraine appear in the best practice frontier more frequently under different direction vector settings.

Fig. 2 shows the boxplots of the three indices for G1 and G2. It can be observed that the medians of the three indices for G1 are all greater than those for G2, while the converse is observed for the variances.

For G1 countries, the average scores of EPI_1, CPI_1 and $ECPI_1$ are 0.76, 0.65 and 0.72, respectively. There are 43 countries having performance scores in $ECPI_1$ higher than the average level. Specifically, Spain and Tunisia have the score of 1 for all the three indices, which implies that they are located at the frontier of best practice. In addition, the EPI_1 scores for South Africa are equal to one while its CPI_1 and $ECPI_1$ scores are lower than one, indicating that it performs relatively better in energy utilization but not in CO_2 emissions. With regard to China, India and South Africa which in 2005 were highly dependent on fossil fuels in electricity generation, they are all found to be below the average in CO_2 emission performance in G1. Considering the energy performance of those countries, China and South Africa performed better than the average level of G1, while India was below the average level.

For G2 countries, the average scores of EPI_2, CPI_2 and $ECPI_2$ are 0.54, 0.41 and 0.44, respectively. At the national level, there are three countries, Switzerland, Lithuania and Ukraine, having EPI_2, CPI_2 and $ECPI_2$ scores equal to one, which implies that they are located on the frontier of the best practice. As the second largest fuel consumer and CO_2 emitter in G2, Russia performed relatively well in energy use and CO_2 emissions and ranks fourth in CPI_2 and $ECPI_2$ and fifth in EPI_2. However, the variances of EPI_2, CPI_2 and $ECPI_2$ for G2 are quite large, which indicates that the energy performance and CO_2 emission performance in electricity generation of countries are significantly different from each other. Specifically, there are 12 countries with EPI_2, CPI_2 and $ECPI_2$ scores no more than 0.2, which could be due to their relatively low generation efficiency and/or coal-dominated fuel mix in electricity generation.

As discussed above, the mean values for the three indices of G1 are all greater than that of G2. However, it is inappropriate to conclude that G1 countries have better energy and CO_2 emission performance than G2 countries since EPI, CPI and ECPI are dependent on the production framework employed and the frameworks for G1 and G2 are different. Nevertheless, the results obtained indicate that the countries in G1 are relatively closer to the frontier of best practice constructed from the countries, which could be explained by the fact that more than 90% of countries in G1 are non-OECD countries with relatively small variations in the input and output variables used.

Next, three hypotheses are proposed and tested to investigate whether there exist significant differences in *EPI*, *CPI* and *ECPI* between OECD and non-OECD countries in G1 and G2, respectively. The proposed null hypotheses are described as follows:

(1) OECD countries have the same energy performance as non-OECD countries in electricity generation;
(2) OECD countries have the same carbon emission performance as non-OECD countries in electricity generation;

[11] The data set used in Ang et al. (2011) was collected or derived from the IEA statistical database http://www.iea.org/stats/index.asp. Our empirical study is therefore based only on the data for a single year. Further research may be performed using time-series data or the data of a more recent year to see if the performance changes over time. The three countries excluded in the present study are Congo, Paraguay and Nepal.

[12] Petajoule (PJ), equal to 10^{15} joules, is a measurement unit of energy consumption. It is equal to 0.2778 TWh or 0.0239 Mtoe.

[13] The values of non-radial directional distance functions and three indices for the remaining countries can be obtained from the corresponding author upon request.

Table 2
Summary of the countries forming the best practice frontier.

G1						G2					
$(-F,E,0)$		$(0,E,-C)$		$(-F,E,-C)$		$(-F,E,H,0)$		$(0,E,H,-C)$		$(-F,E,H,-C)$	
Spain	76	Spain	6	Spain	6	Norway	15	Switzerland	35	Switzerland	36
South Africa	16	Tunisia	81	Tunisia	81	Sweden	10	Lithuania	20	Lithuania	21
Tunisia	41					Switzerland	16	Ukraine	29	Ukraine	29
						Lithuania	14				
						Macedonia	20				
						Mongolia	8				
						Ukraine	26				

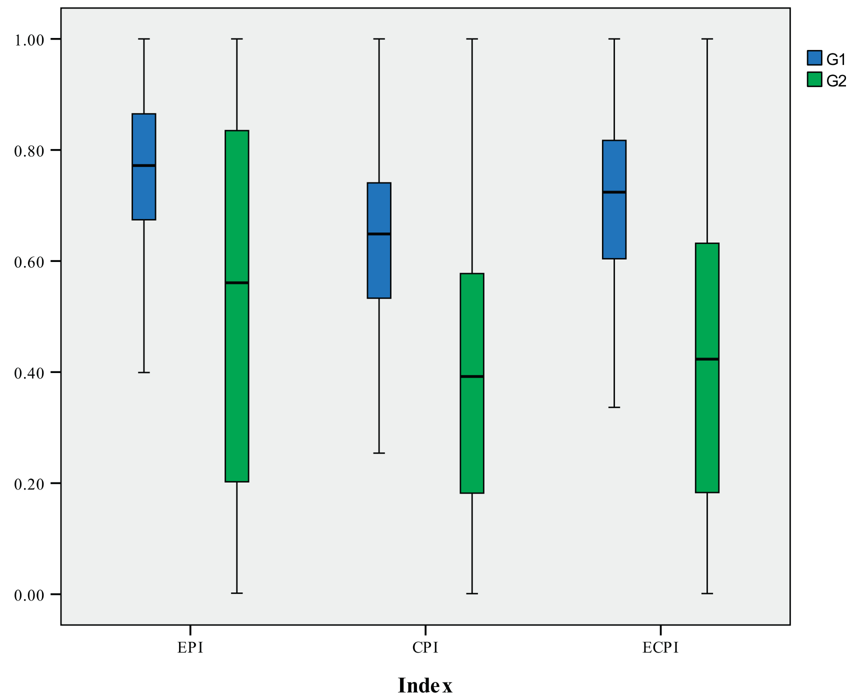

Fig. 2. Boxplots of *EPI*, *CPI* and *ECPI* for G1 and G2.

Table 3
Summary of hypothesis test results.

Hypothesis	Mann–Whitney U	p-Value
H_{0a}: Mean($EPI_{OECD-G1}$) = Mean($EPI_{non-OECD-G1}$)	73.000	0.001
H_{0b}: Mean($CPI_{OECD-G1}$) = Mean($CPI_{non-OECD-G1}$)	120.500	0.009
H_{0c}: Mean($ECPI_{OECD-G1}$) = Mean($ECPI_{non-OECD-G1}$)	97.500	0.003
H_{0d}: Mean($EPI_{OECD-G2}$) = Mean($EPI_{non-OECD-G2}$)	237.000	0.458
H_{0e}: Mean($CPI_{OECD-G2}$) = Mean($CPI_{non-OECD-G2}$)	241.000	0.496
H_{0f}: Mean($ECPI_{OECD-G2}$) = Mean($ECPI_{non-OECD-G2}$)	238.000	0.467

(3) OECD countries have the same integrated energy-carbon performance as non-OECD countries in electricity generation.

As the three indexes derived do not follow normal distribution, we follow the popular practice of statistical testing in DEA by employing the Wilcoxon–Mann–Whitney rank-sum-test to test the three hypotheses. The results obtained are summarized in Table 3.

It can be observed from Table 3 that the three hypotheses for G1 are all rejected at the 0.01 level of significance, implying that OECD countries in G1 outperform non-OECD countries on all the three aspects. On the contrary, there is no statistical evidence for rejection of all the three hypotheses for G2 at the 0.01 level of significance. The results indicate that there is no significant difference in energy, carbon emission and integrated energy-carbon performance between OECD and non-OECD countries in G2. Thus, OECD countries in general seem to have better performance in all the three aspects than non-OECD countries. It would be an indication that non-OECD countries have more potential in reducing energy consumption and CO_2 emissions. Also, the results clearly suggest that developing countries could make such progress by switching to cleaner energy sources, improving energy efficiency and assimilating more advanced energy conservation technology.

We have also investigated the correlation between EPI and aggregate generation efficiency (i.e. the ratio of electricity generation to fossil fuel input) and that between CPI and aggregate carbon intensity (i.e. the ratio of CO_2 emissions to electricity generation). The Pearson and Spearman correlation coefficients obtained are shown in Table 4. It can be observed that there exists a positive correlation between EPI and aggregate generation efficiency, and a negative correlation between CPI and aggregate carbon intensity. It implies that a country with high generation efficiency usually has a better energy performance and a country with low carbon intensity often has a better CO_2 emission performance. Table 3 also shows that the EPI and CPI correlation for G1 is stronger than that for G2, which might be due to the inclusion of heat output in calculating the performance scores of G2 countries. The correlation analysis indicates that the energy and CO_2 emission performance indexes de-

Table 4
Summary of correlation coefficients and Spearman correlation coefficients.

G1			G2		
	EPI_1	Aggregate generation efficiency		EPI_2	Aggregate generation efficiency
EPI_1	1		EPI_2	1	
Aggregate generation efficiency	0.949 (0.947)	1	Aggregate generation efficiency	0.319(0.201)	1
	CPI_1	Aggregate carbon intensity		CPI_2	Aggregate carbon intensity
CPI_1	1		CPI_2	1	
Aggregate carbon intensity	−0.907(−0.996)	1	Aggregate carbon intensity	−0.433 (−0.435)	1

Note: The value in parentheses represents the Spearman correlation coefficient.

Table 5
Summary statistics of the three indexes from non-radial and original directional distance functions.

	Mean value			Standard deviation		
	EPI	*CPI*	*ECPI*	*EPI*	*CPI*	*ECPI*
Non-radial directional distance function	0.6869	0.5619	0.6194	0.2514	0.2365	0.2486
Directional distance function	0.8326	0.6406	0.7856	0.1282	0.1904	0.1508

rived in this study are closely related to aggregate generation efficiency and carbon intensity.

3.3. Comparison with the results from directional distance function

As non-radial directional distance function is an extension to the original directional distance function, it would be meaningful to conduct a comparison between the results obtained from the two approaches. Methodologically, when the original directional distance function is used, the three indexes will collapse to the same form as $(1 - \beta^*)/(1 + \beta^*)$. Table 5 shows the means and standard deviations of the three indexes derived from non-radial and original directional distance functions.

It can be observed from Table 5 that there exist significant differences in the three indexes obtained from the original and non-radial directional distance functions. Generally, the mean values for the three indexes obtained from the original directional distance function are greater than those from non-radial one, while the converse is observed for the standard deviations. It is mainly caused by the fact that the non-radial directional distance function is capable of identifying all the slacks in input and output variables so that the gap between numerator and denominator for each of the three indexes from the non-radial directional distance function approach becomes larger. As the index values for the most inefficient countries become smaller while those for efficient countries are still unity, the non-radial directional distance function approach would certainly produce performance indexes with greater variances.

4. Conclusion

This paper presents a non-radial directional distance function approach to modeling energy and CO_2 emission performance in world electricity generation. We first define and construct the environmental production technologies respectively for countries without and with CHP plants. This grouping allows the construction of the frontier of best practice from homogenous countries and makes the comparison of energy and CO_2 emission performance more consistent. The non-radial directional distance function approach is then proposed and several performance indexes for measuring energy and CO_2 emission performance in electricity generation are created. A major strength of the non-radial directional distance function approach is that it allows for the adjustments of inputs and outputs non-proportionally. Compared to the several recent studies on non-radial directional distance function, e.g. Färe and Grosskopf (2010), Fukuyama and Weber (2010), Fukuyama et al. (2011) and Barros et al. (2012), which directly provide the DEA models for calculating directional slacks-based in efficiency measures, this papers starts from the characterization of the non-radial directional distance function holding some desirable mathematical properties. The DEA models are then constructed from the non-radial directional distance function for calculating inefficiency scores. The proposed approach has been applied to evaluate the energy and CO_2 emission performance of 126 countries in electricity generation.

The empirical study shows that OECD countries outperformed non-OECD countries in carbon and integrated energy-carbon performance, while OECD countries were similar to non-OECD countries in energy performance. Several large countries, such as China, India and the United States are found to have rather poor energy and CO_2 emission performance. This could be explained by their relatively low electricity generation efficiency and the coal-dominated fuel input in electricity generation. It also shows that there exist huge potential in reducing fossil energy consumption and CO_2 emission from electricity generation in these countries. Furthermore, we have also found a significant correlation between EPI and aggregate generation efficiency and between CPI and aggregate carbon intensity.

It should be pointed out the empirical study is only based on the 2005 data and further research may be carried out using time-series data or the data for a more recent year. Although the empirical study conducted in this paper is based on country-level data, the proposed approach can also be directly applied to measure energy and environmental performance of electricity generation at plant level if the data needed are available. In addition to measure energy and CO_2 emission performance of electricity generation with single input and undesirable output, the proposed non-radial directional distance function approach can also be easily adapted in cases, where there are more input and output variables.

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Appendix A. Several important properties of non-radial directional distance function

A.1. Translation property

That is $\overrightarrow{D}_1(F+\alpha g_{1F}, E+\alpha g_{1E}, C+\alpha g_{1C}; g_1) = \overrightarrow{D}_1(F,E,C;g_1) - \alpha$, $\alpha \in \Re$.

Proof.

$$\begin{aligned}\overrightarrow{D}_1&(F+\alpha g_{1F}, E+\alpha g_{1E}, C+\alpha g_{1C}; g_1) = \sup\{\mathbf{w}_1^T\boldsymbol{\beta}_1\\ &: ((F+\alpha g_{1F}, E+\alpha g_{1E}, C+\alpha g_{1C}) + g_1 \cdot diag(\boldsymbol{\beta}_1)) \in T_1\}\\ &= \sup\{\mathbf{w}_1^T\boldsymbol{\beta}_1 : ((F,E,C) + \alpha(g_{1F}, g_{1E}, g_{1C}) + g_1 \cdot diag(\boldsymbol{\beta}_1)) \in T_1\}\end{aligned}$$

Defining $\alpha = (\alpha,\alpha,\alpha)^T$, we then have

$$\begin{aligned}\overrightarrow{D}_1&(F+\alpha g_{1F}, E+\alpha g_{1E}, C+\alpha g_{1C}; g_1)\\ &= \sup\{\mathbf{w}_1^T\boldsymbol{\beta}_1 : ((F,E,C) + g_1 \cdot diag(\boldsymbol{\beta}_1 + \boldsymbol{\alpha})) \in T_1\}\\ &= \sup\{\mathbf{w}_1^T(\boldsymbol{\beta}_1 + \boldsymbol{\alpha}) : ((F,E,C) + g_1 \cdot diag(\boldsymbol{\beta}_1 + \boldsymbol{\alpha})) \in T_1\} - \mathbf{w}_1^T\boldsymbol{\alpha}\\ &= \overrightarrow{D}_1(F,E,C;g_1) - \alpha \quad \square\end{aligned}$$

A.2. Homogenous of degree − 1

In the directional vector $g_1 = (g_{1F}, g_{1E}, g_{1C})$, i.e. $\overrightarrow{D}_1(F,E,C;\lambda g_1) = \lambda^{-1}\overrightarrow{D}_1(F,E,C;g_1), \lambda > 0$.

Proof.

$$\begin{aligned}\overrightarrow{D}_1(F,E,C;\lambda g_1) &= \sup\{\mathbf{w}_1^T\boldsymbol{\beta}_1 : ((F,E,C) + \lambda g_1 \cdot diag(\boldsymbol{\beta}_1)) \in T_1\}\\ &= \lambda^{-1}\sup\{\mathbf{w}_1^T(\lambda\boldsymbol{\beta}_1) : ((F,E,C) + g_1 \cdot diag(\lambda\boldsymbol{\beta}_1)) \in T_1\}\\ &= \lambda^{-1}\overrightarrow{D}_1(F,E,C;\lambda g_1) \quad \square\end{aligned}$$

A.3. Homogenous of degree + 1

In inputs and outputs if the technology exhibits constant returns to scale, i.e. $\overrightarrow{D}_1(\lambda F, \lambda E, \lambda C; g_1) = \lambda\overrightarrow{D}_1(F,E,C;g_1), \lambda > 0$.

Proof.

$$\begin{aligned}\overrightarrow{D}_1(\lambda F, \lambda E, \lambda C; g_1) &= \sup\{\mathbf{w}_1^T\boldsymbol{\beta}_1 : (\lambda(F,E,C) + g_1 \cdot diag(\boldsymbol{\beta}_1)) \in T_1\}\\ &= \sup\{\mathbf{w}_1^T\boldsymbol{\beta}_1 : \lambda((F,E,C) + g_1 \cdot diag(\lambda^{-1}\boldsymbol{\beta}_1)) \in T_1\}\\ &= \lambda\sup\{\mathbf{w}_1^T(\lambda^{-1}\boldsymbol{\beta}_1) : \lambda((F,E,C) + g_1 \cdot diag(\lambda^{-1}\boldsymbol{\beta}_1)) \in T_1\}\\ &= \lambda\sup\{\mathbf{w}_1^T(\lambda^{-1}\boldsymbol{\beta}_1) : ((F,E,C) + g_1 \cdot diag(\lambda^{-1}\boldsymbol{\beta}_1)) \in T_1\}\\ &= \lambda\overrightarrow{D}_1(F,E,C;g_1) \quad \square\end{aligned}$$

Appendix B. Occurrence of multiple optimal solutions for Eqs. (7) and (12)

B.1. Special case of Eq. (7)

Model (B.1) has multiple optimal solutions if its objective function value is equal to 1/3, i.e. $\beta_{1F}^* + \beta_{1E}^* + \beta_{1C}^* = 1$.

$$\begin{aligned}\overrightarrow{D}_1(F,E,C;g_1) = \max\, & 1/3\beta_{1F} + 1/3\beta_{1E} + 1/3\beta_{1C}\\ \text{s.t} & \sum_{n=1}^{N} z_{1n}F_{1n} \leqslant (1-\beta_{1F})F\\ & \sum_{n=1}^{N} z_{1n}E_{1n} \geqslant (1+\beta_{1E})E\\ & \sum_{n=1}^{N} z_{1n}C_{1n} = (1-\beta_{1C})C\\ & z_{1n} \geqslant 0, \quad n = 1,2,\ldots,N\\ & \beta_{1F},\ \beta_{1E},\ \beta_{1C} \geqslant 0\end{aligned} \tag{B.1}$$

Proof. Assume that $(z_{11}^*, \cdots, z_{1N}^*, \beta_{1F}^*, \beta_{1E}^*, \beta_{1C}^*)$ is an optimal solution to Eq. (B.1). Since $\beta_{1F}^* + \beta_{1E}^* + \beta_{1C}^* = 1$, at least one of β_{1F}^*, β_{1E}^* and β_{1C}^* is not equal to zero. Without loss of generality, we suppose that $\beta_{1F}^* \neq 0$. If $\beta_{1F}^* = 1$, the intensity variables in Eq. (B.1), i.e. z_{1n}, must be equal to zero in order to satisfy the first constraint. This, however, will violate the second constraint. Therefore, β_{1F}^* cannot take the value of unity. Therefore, $(z_{11}^*/(1-\beta_{1F}^*), \cdots, z_{1N}^*/(1-\beta_{1F}^*),\ 0, (\beta_{1E}^* + \beta_{1F}^*)/(1-\beta_{1F}^*), (\beta_{1C}^* - \beta_{1F}^*)/(1-\beta_{1F}^*))$ is also an optimal solution if

$$\begin{aligned}1/3(\beta_{1F}^* + \beta_{1E}^* + \beta_{1C}^*) = {} & 1/3(\beta_{1E}^* + \beta_{1F}^*)/(1-\beta_{1F}^*)\\ & + 1/3(\beta_{1C}^* - \beta_{1F}^*)/(1-\beta_{1F}^*)\end{aligned} \tag{B.2}$$

From Eq. (B.2), we can derive that

$$\frac{1/3 \cdot \beta_{1F}^* \cdot (\beta_{1F}^* + \beta_{1E}^* + \beta_{1C}^* - 1)}{1-\beta_{1F}^*} = 0,$$

which is equivalent to $\beta_{1F}^* + \beta_{1E}^* + \beta_{1C}^* = 1$. $\square$

B.2. Special case of Eq. (12)

Model (B.3) has multiple optimal solutions if its objective function value is equal to 1/3, i.e. $\beta_{2F}^* + 1/2\beta_{2E}^* + 1/2\beta_{2H}^* + \beta_{2C}^* = 1$.

$$\begin{aligned}\overrightarrow{D}_2(F,E,H,C;g_2) = \max\, & 1/3\beta_{2F} + 1/6\beta_{2E} + 1/6\beta_{2H} + 1/3\beta_{2C}\\ \text{s.t} & \sum_{m=1}^{M} z_{2m}F_{2m} \leqslant (1-\beta_{2F})F\\ & \sum_{m=1}^{M} z_{2m}E_{2m} \geqslant (1+\beta_{2H})E\\ & \sum_{m=1}^{M} z_{2m}H_{2m} \geqslant (1+\beta_{2H})H\\ & \sum_{m=1}^{M} z_{2m}C_{2m} = (1-\beta_{2C})C\\ & z_{2m} \geqslant 0, \quad m = 1,2,\ldots,M\\ & \beta_{2F},\ \beta_{2E},\ \beta_{2H},\ \beta_{2C} \geqslant 0\end{aligned} \tag{B.3}$$

Proof. Assume that $(z_{21}^*, \ldots, z_{2M}^*, \beta_{2F}^*, \beta_{2E}^*, \beta_{2H}^*, \beta_{2C}^*)$ is an optimal solution to Eq. (B.3). Since $\beta_{2F}^* + \beta_{2E}^* + \beta_{2H}^* + \beta_{2C}^* = 1$, at least one of β_{2F}^*, β_{2E}^*, β_{2H}^* and β_{2C}^* is greater than zero. Without loss of generality, we suppose that $\beta_{2F}^* \neq 0$. Certainly, β_{2F}^* cannot take the value of unity in order not to violate the second and third constraints. Under this case, $(z_{21}^*/(1-\beta_{2F}^*), \ldots, z_{2M}^*/(1-\beta_{2F}^*),\ 0, (\beta_{2E}^* + \beta_{2F}^*)/(1-\beta_{2F}^*), (\beta_{2H}^* + \beta_{2F}^*)/(1-\beta_{2F}^*), (\beta_{2C}^* - \beta_{2F}^*)/(1-\beta_{2F}^*))$ is also an optimal solution if

$$\begin{aligned}&1/3\beta_{2F}^* + 1/6\beta_{2E}^* + 1/6\beta_{2H}^* + 1/3\beta_{2C}^*)\\ &\quad = 1/6(\beta_{2E}^* + \beta_{2F}^*)/(1-\beta_{2F}^*) + 1/6(\beta_{2H}^* + \beta_{2F}^*)/(1-\beta_{2F}^*)\\ &\qquad \times +1/3(\beta_{2C}^* - \beta_{2F}^*)/(1-\beta_{2F}^*)\end{aligned}$$

From Eq. (B.4), we can derive that

$$\frac{2\beta_{2F}^* \cdot (\beta_{2F}^* + 1/2\beta_{2E}^* + 1/2\beta_{2H}^* + \beta_{2C}^* - 1)}{1-\beta_{2F}^*} = 0,$$

which is equivalent to $\beta_{2F}^* + 1/2\beta_{2E}^* + 1/2\beta_{2H}^* + \beta_{2C}^* = 1$. $\square$

Appendix C. Estimates of the non-radial directional distance function and the three indices for selected countries

	G1					G2					
	Country	β_{1F}	β_{1E}	β_{1C}	Index	Country	β_{2F}	β_{2E}	β_{2H}	β_{2C}	Index
EPI	Australia	0.0000	0.1618	–	0.8607	Austria	0.0000	0.0000	1.7246	–	0.5370
	Iceland	0.0731	0.0035	–	0.9237	Belgium	0.0000	0.0000	7.7729	–	0.2046
	Ireland	0.0592	0.0165	–	0.9256	Canada	0.0000	0.0000	14.7956	–	0.1191
	Japan	0.0180	0.0308	–	0.9526	Czech Republic	0.0000	0.0000	0.6639	–	0.7508
	Mexico	0.1251	0.0341	–	0.8460	Denmark	0.0000	0.0000	0.4499	–	0.8163
CPI	Australia	–	0.2387	0.3555	0.5203	Austria	–	0.0000	1.8069	0.0173	0.5163
	Iceland	–	0.0017	0.2142	0.7845	Belgium	–	0.0000	8.1376	0.0216	0.1930
	Ireland	–	0.0000	0.2246	0.7754	Canada	–	0.0000	25.3576	0.1358	0.0632
	Japan	–	0.0000	0.1570	0.8430	Czech Republic	–	0.0000	2.5603	0.2093	0.3468
	Mexico	–	0.0936	0.1674	0.7613	Denmark	–	0.0000	0.5787	0.2725	0.5642
ECPI	Australia	0.1927	0.0000	0.4797	0.6638	Austria	0.0000	0.0000	1.8069	0.0173	0.5208
	Iceland	0.0017	0.0000	0.2155	0.8914	Belgium	0.0000	0.0000	8.1376	0.0216	0.1952
	Ireland	0.0000	0.0000	0.2246	0.8877	Canada	0.0000	0.0000	25.3576	0.1358	0.0681
	Japan	0.0000	0.0000	0.1570	0.9215	Czech Republic	0.0000	0.0000	2.5603	0.2093	0.3927
	Mexico	0.0856	0.0000	0.2387	0.8378	Denmark	0.1829	0.0000	0.0000	0.4039	0.7066

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