Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries: Comment

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In a recent issue of this *Review,* Rolf Färe et al. (FGNZ) (1994) analyzed the rates of productivity growth over the period 1979–1988 in 17 OECD countries. They used Data Envelopment Analysis to measure Malmquist productivity indices for the individual countries by the ratio of the values of the output distance functions for a reference technology exhibiting constant returns to scale (CRS) at the input-output bundles of the same country observed in adjacent years. The Malmquist index is first decomposed into two factors: one showing technical change and the other, changes in technical efficiency, which can be interpreted as "catching up." The "catching up" term is further factored into two terms: one representing pure technical efficiency change and the other, changes in scale efficiency. This extended decomposition conceptualizes a technology characterized by variable returns to scale (VRS).[1] Their use of CRS and VRS within the same decomposition of the Malmquist index raises a problem of internal consistency. Their technical change (TECHCH) measure corresponds to shifts over time in the CRS frontier. The other factors—pure efficiency change (PEFFCH) and scale efficiency change (SCH)—are derived from VRS frontiers from two different periods, however. If CRS is assumed to hold, the TECHCH term correctly shows the shift in the frontier. But, under CRS no scale effect exists at all. Hence, the extended decomposition is misleading. On the other hand, if the VRS assumption is correct, FGNZ's TECHCH *does not show how the maximum producible output changes due to technical change holding the input bundle constant.* In other words, it does not measure the autonomous shift in the frontier. As we show below, the Malmquist productivity index is correctly measured by the ratio of CRS distance functions even when the technology exhibits variable returns to scale. Thus, we measure the productivity index itself the same way as FGNZ do. There are alternative ways to decompose the same Malmquist index, which, in empirical applications, lead to different conclusions about technical change and efficiency change experienced by individual countries. We propose a decomposition using a VRS frontier as the benchmark. We measure technical change by the ratio of VRS distance functions. While this affects the measured value of the scale efficiency change, the pure technical efficiency change measure remains unaffected. In our empirical application, we use data from an updated version of the Penn World Tables (PWT 5.6).[2] This allows the use of two additional years—1989 and 1990. In the remainder of this paper, we highlight the problem of internal consistency in FGNZ's decomposition and propose an alternative procedure. The empirical application demonstrates how consistent use of the VRS assumption leads one to significantly different conclusions.

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[1] Decomposition of the Malmquist productivity index as defined by Douglas W. Caves et al. (1982a, b) into technical change and "catching up" was introduced by Färe et al. (1992). The extended decomposition is due to Färe et al. (1994).

[2] The Penn World Tables (PWT) were constructed by Robert Summers and Alan Heston, who have periodically updated these tables. Currently, the latest version of the PWT can be accessed through the internet from the public domain library of the National Bureau of Economic Research (NBER) at Harvard. Summers and Heston (1991) provides a discussion of how the PWT data were constructed.

I. The Nonparametric Methodology

Consider, for simplicity, a single output-single input industry. Let x_k^t and y_k^t represent the input and output quantities of firm k at time t. The average productivity (AP)[3] of this firm at time t is

$$(1) \qquad \mathrm{AP}_k^t = y_k^t / x_k^t.$$

Thus, a productivity index for this firm at time $t + 1$, with period t treated as the base, will be

$$(2) \qquad \Pi_k = \mathrm{AP}_k^{t+1} / \mathrm{AP}_k^t = (y_k^{t+1} / x_k^{t+1}) / (y_k^t / x_k^t).$$

This productivity index itself does not in any way depend on assumptions about returns to scale. In order to identify the sources of productivity change, however, we need a benchmark technology. The returns-to-scale assumptions become important in the definition of the benchmark technology. This is shown diagrammatically in Figure 1. Consider an industry consisting of four firms—A, B, C, and D. Points A_0 through D_0 in Figure 1 show the observed input-output levels of these firms in period 0. Similarly, A_1 through D_1 show their input-output levels in period 1. Firm A uses input $0x_0$ to produce output A_0x_0 in period 0 and input $0x_1$ to produce output A_1x_1 in period 1. Thus, the productivity index for firm A in period 1 is

$$(3) \qquad \Pi_A = (A_1x_1 / 0x_1) / (A_0x_0 / 0x_0).$$

By convexity, all points in the convex hull of the points A_0, B_0, C_0, and D_0 (i.e., all convex combinations of these points) represent feasible input-output combinations in period 0. The free disposal convex hull is the set of points bounded by the horizontal axis and the broken line $E_0B_0C_0D_0$-extension. Under VRS, all points in this region represent feasible input-output combinations in period 0. Under CRS, however, all radial expansion and (nonnegative) contraction of feasible input-output bundles are also feasible. Thus, the CRS production possibility set in period 0 is the cone formed by the horizontal axis and the ray $0R_0$ through the point C_0. Similarly, the VRS frontier in period 1 is the broken line $E_1B_1C_1D_1$-extension and the CRS frontier is the ray $0R_1$ through the point C_1. In period 0, the maximum producible output from input $0x_0$ is P_0x_0 under the CRS assumption and T_0x_0 under the VRS assumption. The distance functions are

$$(4) \qquad D_c^0(x_0, y_0) = A_0x_0 / P_0x_0$$

$$D_c^0(x_1, y_1) = A_1x_1 / P_1x_1$$

under CRS, and

$$(5) \qquad D_v^0(x_0, y_0) = A_0x_0 / T_0x_0$$

$$D_v^0(x_1, y_1) = A_1x_1 / T_1x_1$$

under VRS. The productivity index of firm A can be expressed alternatively

$$(6) \qquad \Pi_A^0 = D_c^0(x_1, y_1) / D_c^0(x_0, y_0),$$

$$(7) \qquad \Pi_A^1 = D_c^1(x_1, y_1) / D_c^1(x_0, y_0).$$

Clearly, the productivity index is equivalent to the ratio of the CRS distance functions *even if the technology was not characterized by constant returns to scale.* Compare, now, the CRS and the VRS frontiers in period 0. Along the CRS frontier, the average productivity remains constant. But this is not the case along the VRS frontier. Both T_0 and T_1 are points on the frontier and are, therefore, technically efficient. Average productivity at T_0, however, is higher than the average productivity at T_1. In fact, the point of highest average productivity along the VRS frontier in period 0 is C_0. This corresponds to what Rajiv D. Banker (1984) and Banker et al. (1984) call the most productive scale size (MPSS). The average productivity at the MPSS of the VRS frontier is equal to the constant average productivity at any point on the CRS frontier. The scale efficiency at any

[3] In the multiple-input, multiple-output case the concept of average productivity does not apply. Hence all of our results cannot necessarily be generalized.

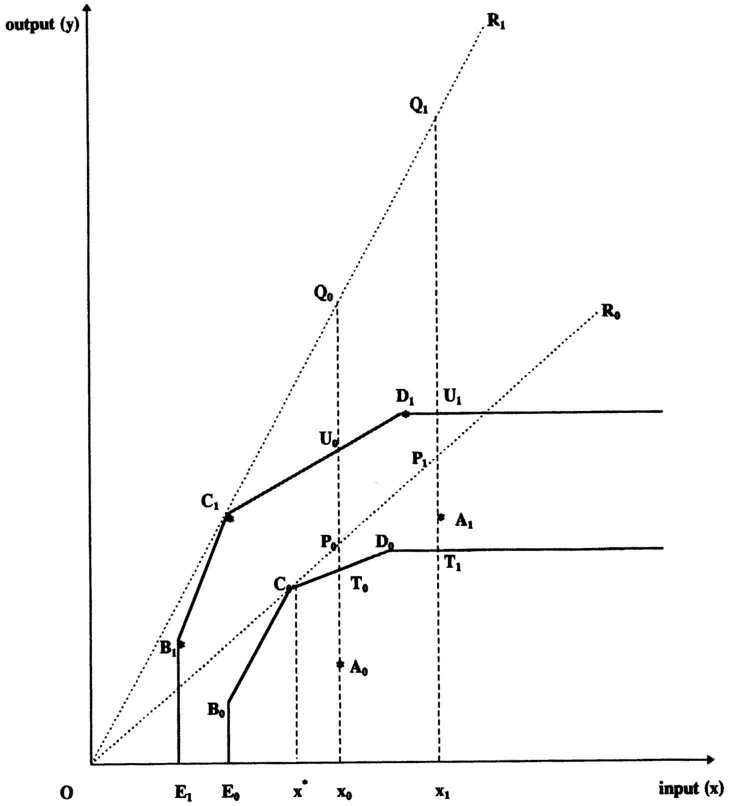

FIGURE 1. THE VRS AND CRS PRODUCTION POSSIBILITY SETS

point on the frontier is measured by the ratio of the average productivity at that point to the average productivity at the MPSS. Thus,

$$(8)\quad SE^0(x_0,y_0) = D_c^0(x_0,y_0)\,/\,D_v^0(x_0,y_0),$$

$$(9)\quad SE^0(x_1,y_1) = D_c^0(x_1,y_1)\,/\,D_v^0(x_1,y_1).$$

Hence, the productivity index, Π_A, can be expressed alternatively as

$$(10)\quad \Pi_A^0 = \frac{D_v^0(x_1,y_1)}{D_v^0(x_0,y_0)}\frac{SE^0(x_1,y_1)}{SE^0(x_0,y_0)},$$

$$(11)\quad \Pi_A^1 = \frac{D_v^1(x_1,y_1)}{D_v^1(x_0,y_1)}\frac{SE^1(x_1,y_1)}{SE^1(x_0,y_0)}.$$

Using the geometric mean,

$$(12)\quad \Pi_A = \left[\frac{D_v^0(x_1,y_1)}{D_v^0(x_0,y_0)}\frac{D_v^1(x_1,y_1)}{D_v^1(x_0,y_0)}\right]^{1/2} \times \left[\frac{SE^0(x_1,y_1)}{SE^0(x_0,y_0)}\frac{SE^1(x_1,y_1)}{SE^1(x_0,y_0)}\right]^{1/2}.$$

The first factor on the right-hand side can be further decomposed as

$$\left[\frac{D_v^0(x_1,y_1)}{D_v^0(x_0,y_0)}\frac{D_v^1(x_1,y_1)}{D_v^1(x_0,y_0)}\right]^{1/2} = \left[\frac{D_v^0(x_0,y_0)}{D_v^1(x_0,y_0)}\cdot\frac{D_v^0(x_1,y_1)}{D_v^1(x_1,y_1)}\right]^{1/2} \times \frac{D_v^1(x_1,y_1)}{D_v^0(x_0,y_0)}.$$

Thus,

$$(13)\quad \Pi_A = (\text{TECHCH}(v)) \times (\text{PEFFCH})\cdot(\text{SCH}(v)),$$

where

$$(14)\quad \text{TECHCH}(v) = \left[\frac{D_v^0(x_0, y_0)}{D_v^1(x_0, y_0)}\cdot\frac{D_v^0(x_1, y_1)}{D_v^1(x_1, y_1)}\right]^{1/2},$$

$$(15)\quad \text{PEFFCH} = \frac{D_v^1(x_1, y_1)}{D_v^0(x_0, y_0)},$$

$$(16)\quad \text{SCH}(v) = \left[\frac{\text{SE}^0(x_1, y_1)}{\text{SE}^0(x_0, y_0)}\cdot\frac{\text{SE}^1(x_1, y_1)}{\text{SE}^1(x_0, y_0)}\right]^{1/2}.$$

This decomposition of the Malmquist productivity index is quite different from the extended decomposition performed by FGNZ.[4] The only factor which is identical is PEFFCH. As for the technical change factor, ours is a geometric mean of the ratios of VRS distance functions whereas FGNZ measure technical change by the geometric mean of the ratios of CRS distance functions. Also, the other factor relating to scale efficiency change differs in the two decompositions. FGNZ's scale change factor (SCH) is simply the ratio of the scale efficiencies of the bundles (x_0, y_0) and (x_1, y_1) using own-period VRS technologies as the benchmark. Our measure, SCH(v), is a geometric mean of the ratios of scale efficiencies of the two bundles using in turn the VRS technologies from the two periods as the benchmark. In that sense, it is more in the spirit of a Fisher index.

A closer look at Figure 1 will reveal why the extended decomposition performed by FGNZ is not internally consistent. In standard theory technical change is measured by the autonomous shift in the production function over time *holding the input bundle constant* (Robert G. Chambers, 1988 p. 205). FGNZ's measure of technical change is

$$(17)\quad \text{TECHCH} = \left[\frac{Q_0x_0}{P_0x_0}\cdot\frac{Q_1x_1}{P_1x_1}\right]^{1/2}.$$

This correctly measures technical change when constant returns to scale holds. But in that case, the VRS frontiers do not characterize the technologies in the two periods any more. In fact, the points T_0 and T_1 are *interior points* in period 0. Similarly, points U_0 and U_1 are interior points in period 1. Obviously, under CRS there is, by definition, no scale inefficiency. Thus the decomposition of the "catch up" factor into pure efficiency change and scale efficiency change is inappropriate.

Now assume, instead, that VRS (rather than CRS) holds. In this case, the rate of technical change at the input bundle x_0 is measured by the shift of the VRS frontier from T_0 to U_0 and *not by the shift in the CRS frontier* from P_0 to Q_0. Similar reasoning applies to the measurement of technical change at the input level x_1. By looking at the shift in the CRS frontier, FGNZ are actually comparing point C_0 on the VRS frontier in period 0 with point C_1 on the VRS frontier in period 1. Because these two points correspond to two different input levels, differences in the output levels between them reflect not only technical change but also returns-to-scale effect. On the other hand,

$$(18)\quad \text{TECHCH}(v) = \left[\frac{U_0x_0}{T_0x_0}\cdot\frac{U_1x_1}{T_1x_1}\right]^{1/2}$$

is a geometric mean of the shifts in the VRS frontier at the two input levels and correctly measures technical change between the two periods.

It should be emphasized here that although the scale efficiency term involves both CRS and VRS distance functions, it only uses a point like P_0 on what would have been the CRS *frontier* merely as an artifact in order to measure the difference in average productivity between the points T_0 and C_0, both of which are on the VRS frontier. We do not need to assume that a point like P_0 is feasible. Point

[4] C. A. Knox Lovell and Emili Grifell-Tatjé (1994) derived this decomposition in a different way. However, they call this a *generalized* Malmquist index.

TABLE 1—MALMQUIST PRODUCTIVITY INDEX AND ITS DECOMPOSITION

	Annual averages (1979–1990)			
Country	Malmquist index	Technical change index	Technical efficiency index	Scale efficiency index
Australia	0.97823	0.97576	0.99903	1.00350
Austria	0.96045	0.99328	0.99949	0.96744
Belgium	1.02768	1.02488	1.00295	0.99978
Canada	1.00537	1.02880	1.00361	0.97371
Denmark	1.01436	0.95548	0.99755	1.03183
Finland	1.03204	1.01939	1.01070	1.00169
France	0.99884	0.99839	1.00113	0.99932
Germany	1.03187	1.00426	0.99659	1.03101
Greece	1.00036	1.00120	0.99806	1.00111
Ireland	0.97026	infeasible solution	1.00000	infeasible solution
Italy	0.97862	1.00501	1.00481	0.96908
Japan	1.00981	1.01340	1.00030	0.99616
Norway	1.01095	1.03674	1.0000	0.97513
Spain	0.97562	1.01210	0.99704	0.96681
Sweden	0.98193	0.98588	0.99998	0.99601
United Kingdom	1.00811	0.99830	1.00000	1.00983
United States	0.94445	1.00576	1.00000	0.93904
Sample average[a]	0.99582	1.00367	1.00066	0.99134

[a] The sample mean is the geometric mean.

C_0 lying on the VRS frontier is feasible, however. Because our analysis assumes VRS throughout, the technical change factor is quite consistent with pure efficiency change and our scale change factor.

The own- and cross-period output-oriented distance functions can be obtained by solving the appropriate linear programming problems specified by FGNZ [their problem (17) on page 75]. It should be noted that under the VRS assumption, some of the cross-period problems may not have feasible solutions.[5] This is a limitation of the DEA approach and not of the proposed decomposition. If one econometrically estimates an appropriately specified parametric form of the distance functions, the problem of infeasibility should not arise.[6]

II. The Empirical Analysis

Like FGNZ (1994), we also consider a single output-two input production technology and treat each of the 17 OECD countries in the sample as an individual decision-making unit. Output is measured by the real GDP of a country

[5] In our application, this happens for Ireland. The problem of infeasibility is addressed in details by Ray and Kankana Mukherjee (1996).

[6] For example, we used Dennis Aigner and S. F. Chu's (1968) linear programming approach to estimate translog production frontiers for 1983 and 1984 under VRS assumption. For these two years the cross-period technical efficiency levels for Ireland were 0.9693 when the 1983 data were used against the 1984 frontier, and 1.0319 when 1984 data were evaluated against the 1983 frontier.

TABLE 2—MALMQUIST PRODUCTIVITY INDEX AND ITS FGNZ DECOMPOSITION

	Annual averages (1979–1990)			
Country	Malmquist index	Technical change index	Technical efficiency index	Scale efficiency index
Australia	0.97823	0.97775	0.99903	1.00169
Austria	0.96045	0.96456	0.99949	0.99624
Belgium	1.02768	1.02273	1.00295	1.00188
Canada	1.00537	1.00088	1.00361	1.00087
Denmark	1.01436	1.00999	0.99755	1.00679
Finland	1.03204	1.01663	1.01070	1.00441
France	0.99884	0.99748	1.00113	1.00023
Germany	1.03187	1.03516	0.99659	1.00024
Greece	1.00036	1.00082	0.99806	1.00148
Ireland	0.97026	0.96390	1.00000	1.00659
Italy	0.97862	0.97380	1.00481	1.00014
Japan	1.00981	1.00127	1.00030	1.00822
Norway	1.01095	1.00605	1.00000	1.00488
Spain	0.97562	0.97822	0.99704	1.00030
Sweden	0.98193	0.97956	0.99998	1.00245
United Kingdom	1.00811	1.00810	1.00000	1.00000
United States	0.94445	0.94454	1.00000	1.00000
Sample average[a]	0.99582	0.99303	1.00066	1.00214

[a] The sample mean is the geometric mean.

in U.S. dollars. The two inputs are labor and capital. This study covers the period 1979–1990. Table 1 reports the annual (geometric) averages of the year-to-year Malmquist productivity indexes along with the VRS-based decomposition. For comparison, we show the decomposition following FGNZ in Table 2. The overall average of the CRS-based technical change index was 0.99303. This implies a very slight rate of technical regress. On the other hand, the average of VRS-based technical change was 1.00367 showing technical progress at the rate of 0.37 percent per year. Based on shifts in the CRS frontier one would conclude that the United States experienced technical regress at the rate of 5.5 percent annually. But the VRS decomposition shows technical progress at the rate of 0.58 percent per year. Similarly, for Italy and Spain, the FGNZ decomposition shows technical regress at the rate of over 2 percent per annum. But the VRS-based decomposition shows that both countries experienced technical progress—Italy at a nominal rate of 0.5 percent and Spain at a more noticeable rate of 1.21 percent per year. Also, Table 1 shows that scale efficiency declined, while Table 2 points toward an increase in scale efficiency for these countries. In the case of Denmark, Table 1 shows technical regress, while technical progress is found from Table 2. For Canada, while both tables show technical progress, the VRS-based decomposition shows a much higher annual rate (2.88 percent) than the CRS-based decomposition (0.88 percent). For Japan, also the rate of technical progress is found to be much higher (1.3 percent) under the VRS assumption compared to a modest 0.12 percent under CRS assumption.

A country contributes to an outward shift in the world frontier only if its observed input-output combination lies: (a) outside the frontier for the previous period, and (b) on the frontier for the current period. By FGNZ's criterion, only the United Kingdom and the United States are found to have pushed the frontier forward over the sample period. Our approach, on the other hand, shows that in several years Norway, in conjunction with the United Kingdom and/or the United States, has contributed positively to technical progress. Moreover, in the very last period, Norway alone accounted for the shift in the frontier.

III. Conclusion

In this paper we provide an alternative decomposition of the Malmquist productivity index which avoids the problem of internal consistency encountered in FGNZ's extended decomposition. Remarkably different conclusions follow when one consistently uses a VRS technology as a benchmark.

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