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Economics Letters 88 (2005) 266–271

economics letters

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A global Malmquist productivity index

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Received 2 June 2004; received in revised form 24 January 2005; accepted 16 February 2005
Available online 23 May 2005

Abstract

The geometric mean Malmquist productivity index is not circular, and its adjacent period components can provide different measures of productivity change. We propose a global Malmquist productivity index that is circular, and that gives a single measure of productivity change.

Keywords: Malmquist productivity index; Circularity

JEL classification: C43; D24; O47

1. Introduction

The geometric mean form of the contemporaneous Malmquist productivity index, introduced by Caves et al. (1982), is not circular. Whether this is a serious problem depends on the powers of persuasion of Fisher (1922), who dismissed the test, and Frisch (1936), who endorsed it. The index averages two possibly disparate measures of productivity change. Färe and Grosskopf (1996) state sufficient conditions on the adjacent period technologies for the index to satisfy circularity, and to average the same measures of productivity change. When linear programming techniques are used to compute and decompose the index, infeasibility can occur. Whether this is a serious problem depends on

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0165-1765/$ - see front matter
doi:10.1016/j.econlet.2005.02.013

the structure of the data. Xue and Harker (2002) provide necessary and sufficient conditions on the data for LP infeasibility not to occur.

We demonstrate that the source of all three problems is the specification of adjacent period technologies in the construction of the index. We show that it is possible to specify a base period technology in a way that solves all three problems, without having to impose restrictive conditions on either the technologies or the data.

Berg et al. (1992) proposed an index that compares adjacent period data using technology from a base period. This index satisfies circularity and generates a single measure of productivity change, but it pays for circularity with base period dependence, and it remains susceptible to LP infeasibility.

Shestalova (2003) proposed an index having as its base a sequential technology formed from data of all producers in all periods up to and including the two periods being compared. This index is immune to LP infeasibility, and it generates a single measure of productivity change, but it fails circularity and it precludes technical regress.

Thus no currently available Malmquist productivity index solves all three problems. We propose a new global index with technology formed from data of all producers in all periods. This index satisfies circularity, it generates a single measure of productivity change, it allows technical regress, and it is immune to LP infeasibility.

In Section 2 we introduce and decompose the circular global index. Its efficiency change component is the same as that of the contemporaneous index, but its technical change component is new. In Section 3 we relate it to the contemporaneous index. In Section 4 we provide an empirical illustration. Section 5 concludes.

2. The global Malmquist productivity index

Consider a panel of $i=1,\ldots,I$ producers and $t=1,\ldots,T$ time periods. Producers use inputs $x\in R_{+}^{N}$ to produce outputs $y\in R_{+}^{P}$. We define two technologies. A *contemporaneous* benchmark technology is defined as $T_{c}^{t}=\{(x^{t},y^{t})|x^{t}$ can produce $y^{t}\}$ with $\lambda T_{c}^{t}=T_{c}^{t}$, $t=1,\ldots,T$, $\lambda>0$. A *global* benchmark technology is defined as $T_{c}^{G}=\text{conv}\ \{T_{c}^{1}\cup\ldots\cup T_{c}^{T}\}$. The subscript "c" indicates that both benchmark technologies satisfy constant returns to scale.

A contemporaneous Malmquist productivity index is defined on T_{c}^{s} as

$$M_{c}^{s}\left(x^{t},y^{t},x^{t+1},y^{t+1}\right)=\frac{D_{c}^{s}(x^{t+1},y^{t+1})}{D_{c}^{s}(x^{t},y^{t})}, \tag{1}$$

where the output distance functions $D_{c}^{s}(x,y)=\min\{\phi>0|(x,y/\phi)\in T_{c}^{s}\}$, $s=t,\ t+1$. Since $M_{c}^{t}(x^{t},y^{t},x^{t+1},y^{t+1})\neq M_{c}^{t+1}(x^{t},y^{t},x^{t+1},y^{t+1})$ without restrictions on the two technologies, the contemporaneous index is typically defined in geometric mean form as $M_{c}(x^{t},y^{t},x^{t+1},y^{t+1})=[M_{c}^{t}(x^{t},y^{t},x^{t+1},y^{t+1})\times M_{c}^{t+1}(x^{t},y^{t},x^{t+1},y^{t+1})]^{1/2}$.

A global Malmquist productivity index is defined on T_{c}^{G} as

$$M_{c}^{G}\left(x^{t},y^{t},x^{t+1},y^{t+1}\right)=\frac{D_{c}^{G}(x^{t+1},y^{t+1})}{D_{c}^{G}(x^{t},y^{t})}, \tag{2}$$

where the output distance functions $D_{c}^{G}(x,y)=\min\{\phi>0|(x,y/\phi)\in T_{c}^{G}\}$.

Both indexes compare (x^{t+1},y^{t+1}) to (x^t,y^t), but they use different benchmarks. Since there is only one global benchmark technology, there is no need to resort to the geometric mean convention when defining the global index.

M_c^G decomposes as

$$\begin{aligned}M_c^G\left(x^t,y^t,x^{t+1},\gtreqless y^{t+1}\right) &= \frac{D_c^{t+1}(x^{t+1},y^{t+1})}{D_c^t(x^t,y^t)}\times\left\{\frac{D_c^G(x^{t+1},y^{t+1})}{D_c^{t+1}(x^{t+1},y^{t+1})}\times\frac{D_c^t(x^t,y^t)}{D_c^G(x^t,y^t)}\right\}\\ &= \frac{\mathrm{TE}_c^{t+1}(x^{t+1},y^{t+1})}{\mathrm{TE}_c^t(x^t,y^t)}\times\left\{\frac{D_c^G\left(x^{t+1},y^{t+1}/D_c^{t+1}(x^{t+1},y^{t+1})\right)}{D_c^G\left(x^t,y^t/D_c^t(x^t,y^t)\right)}\right\}\\ &= \mathrm{EC}_c\times\left\{\frac{\mathrm{BPG}_c^{G,t+1}(x^{t+1},y^{t+1})}{\mathrm{BPG}_c^{G,t}(x^t,y^t)}\right\} = \mathrm{EC}_c\times\mathrm{BPC}_c,\end{aligned}\tag{3}$$

where EC_c is the usual efficiency change indicator and $\mathrm{BPG}_c^{G,s}\leq 1$ is a best practice gap between T_c^G and T_c^s measured along rays (x^s,y^s), $s=t$, $t+1$. BPC_c is the change in BPG_c, and provides a new measure of technical change. $\mathrm{BPC}_c\gtreqless 1$ indicates whether the benchmark technology in period $t+1$ in the region $[(x^{t+1},y^{t+1}/D_c^{t+1}(x^{t+1},y^{t+1}))]$ is closer to or farther away from the global benchmark technology than is the benchmark technology in period t in the region $[(x^t,y^t/D_c^t(x^t,y^t))]$.

M_c^G has four virtues. First, like any fixed base index, M_c^G is circular, and since EC_c is circular, so is BPC_c. Second, each provides a single measure, with no need to take the geometric mean of disparate adjacent period measures. Third, but not shown here, the decomposition in (3) can be extended to generate a three-way decomposition that is structurally identical to the Ray and Desli (1997) decomposition of the contemporaneous index. M_c^G and M_c share a common efficiency change component, but they have different technical change and scale components, and so $M_c^G\neq M_c$ without restrictions on the technologies. Finally, the technical change and scale components of M_c^G are immune to the LP infeasibility problem that plagues these components of M_c.

3. Comparing the global and contemporaneous indexes

The ratio

$$\begin{aligned}M_c^G/M_c &= \left[\left(M_c^G/M_c^{t+1}\right)\times\left(M_c^G/M_c^t\right)\right]^{1/2}\\ &= \left\{\left[\frac{D_c^G\left(x^{t+1},y^{t+1}/D_c^{t+1}(x^{t+1},y^{t+1})\right)}{D_c^G\left(x^t,y^t/D_c^{t+1}(x^t,y^t)\right)}\right]\times\left[\frac{D_c^G\left(x^{t+1},y^{t+1}/D_c^t(x^{t+1},y^{t+1})\right)}{D_c^G\left(x^t,y^t/D_c^t(x^t,y^t)\right)}\right]\right\}^{1/2}\\ &= \left\{\left[\frac{\mathrm{BPG}_c^{G,t+1}(x^{t+1},y^{t+1})}{\mathrm{BPG}_c^{G,t+1}(x^t,y^t)}\right]\times\left[\frac{\mathrm{BPG}_c^{G,t}(x^{t+1},y^{t+1})}{\mathrm{BPG}_c^{G,t}(x^t,y^t)}\right]\right\}^{1/2}\end{aligned}\tag{4}$$

is the geometric mean of two terms, each being a ratio of benchmark technology gaps along different rays. $M_c^G/M_c\gtreqless 1$ as projections onto T_c^t and T_c^{t+1} of period $t+1$ data are closer to, equidistant from, or farther away from T_c^G than projections onto T_c^t and T_c^{t+1} of period t data are.

Table 1
Electricity generation data, annual means

	1977	1982	1987	1992
Output (000 MW h)	13,700	13,860	16,180	17,270
Labor (# FTE)	1373	1797	1995	2021
Fuel (billion BTU)	1288	1441	1667	1824
Capital (Törnqvist)	44,756	211,622	371,041	396,386

$M_c^G = M_c$ if $BPG_c^{G,s}(x^{t+1}, y^{t+1}) = BPG_c^{G,s}(x^t, y^t)$, $s = t, t+1$. From the first equality in (4), this condition is equivalent to the condition $M_c^G = M_c^s$, $s = t, t+1$. If this condition holds for all s, it is equivalent to the condition $M_c^t = M_c^1$ for all t. Althin (2001) has shown that a sufficient condition for base period independence is that technical change be Hicks output-neutral (HON). Hence HON is also sufficient for $M_c^G = M_c$.

4. An empirical illustration

We summarize an application intended to illustrate the behavior of M_c^G, and to compare its performance with that of M_c. We analyze a panel of 93 US electricity generating firms in four years (1977, 1982, 1997, 1992). The firms use labor (FTE employees), fuel (BTUs of energy) and capital (a multilateral Törnqvist index) to generate electricity (net generation in MW h). The data are summarized in Table 1. Electricity generation increased by proportionately less than each input did. The main cause of the rapid increase in the capital input was the enactment of environmental regulations mandating the installation of pollution abatement equipment. We are unable to disaggregate the capital input into its productive and abatement components.

Empirical findings are summarized in Table 2. The first three rows report decomposition (3) of M_c^G, and the final three rows report M_c and its two adjacent period components. Columns correspond to time periods.

M_c^G shows a large productivity decline from 1977 to 1982, followed by weak productivity growth. Cumulative productivity in 1992 was 25% lower than in 1977. M_c^G calculated using 1992 and 1977 data generates the same value, verifying that it is circular.

The efficiency change component EC_c of M_c^G (and M_c) is also circular, and cumulates to an 18% improvement. Best practice change, BPC_c, is also circular, and declined by 35%. Capital investment in

Table 2
Global and contemporaneous Malmquist productivity indexes

	1977–1982	1982–1987	1987–1992	Cumulative productivity	1977–1992
M_c^G	0.685	1.064	1.039	0.757	0.757
EC_c	1.163	1.089	0.929	1.176	1.176
BPC_c	0.589	0.977	1.118	0.644	0.644
M_c	0.431	0.895	1.039	0.400	0.592
M_c^t	0.713	0.902	1.053	0.678	1.333
M_c^{t+1}	0.260	0.887	1.024	0.236	0.263

pollution abatement equipment generated cleaner air but not more electricity. Consequently catching up with deteriorating best practice was relatively easy.

Turning to the contemporaneous index M_c reported in the final three rows, the story is not so clear. Cumulative productivity in 1992 was 60% lower than in 1977. However calculating M_c using 1992 and 1977 data generates a smaller 40% decline, verifying that M_c is not circular. Neither figure is close to the 25% decline reported by M_c^G, verifying that technical change was not HON, but (pollution abatement) capital-using. The lack of circularity is reflected in the frequently large differences between M_c^t and M_c^{t+1}, which give conflicting signals when computed using 1992 and 1977 data, with M_c^t signaling productivity growth and M_c^{t+1} signaling productivity decline. Although not reported in Table 2, we have calculated three-way decompositions of M_c^G and M_c. All three components of M_c^G are circular, and LP infeasibility does not occur. In contrast, the technical change and scale components of M_c are not circular, and infeasibility occurs for 13 observations.

The circular global index M_c^G tells a single story about productivity change, and its decomposition is intuitively appealing in light of what we know about the industry during the period. Lacking circularity, M_c and its two adjacent period components tell different stories that are often contradictory. The differences between M_c^G and M_c are a consequence of the capital-using bias of technical change, which was regressive due to the mandated installation of pollution abatement equipment, augmented perhaps by the rate base padding that was prevalent during the period.

5. Conclusions

The contemporaneous Malmquist productivity index is not circular, its adjacent period components can give conflicting signals, and it is susceptible to LP infeasibility. The global Malmquist productivity index and each of its components is circular, it provides single measures of productivity change and its components, and it is immune to LP infeasibility. The global index decomposes into the same sources of productivity change as the contemporaneous index does. A sufficient condition for equality of the two indexes, and their respective components, is Hicks output neutrality of technical change.

The global index must be recomputed when a new time period is incorporated. Diewert's (1987) assertion that "...economic history has to be rewritten..." when new data are incorporated is the base period dependency problem revisited. The problem can be serious when using base periods $t=1$ and $t=T$, but it is likely to be benign when using global base periods $\{1,\ldots,T\}$ and $\{1,\ldots,T+1\}$. While new data may change the global frontier, the rewriting of history is likely to be quantitative rather than qualitative.

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