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A global Malmquist-Luenberger productivity index

Dong-hyun Oh

Abstract This paper introduces an alternative environmentally sensitive productivity growth index, which is circular and free from the infeasibility problem. In doing so, we integrated the concept of the global production possibility set and the directional distance function. Like the conventional Malmquist-Luenberger productivity index, it can also be decomposed into sources of productivity growth. The suggested index is employed in analyzing 26 OECD countries for the period 1990–2003. We also employed the conventional Malmquist-Luenberger productivity index, the global Malmquist productivity index and the conventional Malmquist productivity index for comparative purposes in this empirical investigation.

Keywords Global Malmquist-Luenberger productivity index · Circularity environmentally sensitive productivity growth · Directional distance function

JEL Classification D24 · C61 · O57 · Q43 · Q56

1 Introduction

Although productivity is not the only measure of economic prosperity, standard of living and the competitiveness of an economy, it has been widely recognized as an indirect measure in recent decades (Lall et al. 2002). As international concerns increase about the sustainable growth, recent attempts to develop measures of productivity growth incorporate the negative effect of environmentally harmful by-products. The motivation for these recent developments is that productivity measures are often biased if measured without the environmental effect. To solve this problem the Malmquist productivity index (hereafter, M index) was modified by Chung et al. (1997) to measure *environmentally* sensitive productivity growth, which was named the Malmquist-Luenberger productivity index (hereafter, ML index). The ML index integrates the concepts of the Malmquist productivity index and a directional distance function. Ever since this seminal work, the ML index has been widely used to measure the performance of a wide range of decision-making units (DMUs), such as manufacturing industries (Färe et al. 2001), the public sector (Yu et al. 2008) and countries (Kumar 2006; Yörük and Zaim 2005).

The geometric mean form of the ML index, however, has weakness: it is not circular and it faces a potential linear programming infeasibility problem in measuring cross-period directional distance functions (DDFs). This paper attempts to resolve these problems of the conventional ML index by employing concepts of the global Malmquist productivity growth index of Pastor and Lovell (2005) and the DDF of Luenberger (1992).

In the typical M index, efforts have been made to solve the circularity problem of the geometric mean form.[1] Balk (1998) proves that Hick-neutral technical change is a

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[1] The M index does not take into account the effect of environmentally harmful by-products in general. When using the M index in empirical investigation of this paper, we did not include environmentally harmful by-products.

necessary and sufficient condition for the circularity of the M index. Pastor and Lovell (2007) alleviate this strict condition. They prove that firm-specific time-neutrality of technical change is the necessary and sufficient condition for the circularity of the M index.

Even though the aforementioned conditions are *theoretically* possible for solving the circularity of the M index, it is still hard to employ them in empirical studies. To overcome this weakness, alternative versions of the M index have been developed. Pastor and Lovell (2005) develop the global Malmquist productivity growth index (hereafter, GM index).[2] Asmild and Tam (2007) employ the concept of *global* to calculate productivity growth of a population as a whole. Oh and Lee (2010) propose the metafrontier Malmquist productivity growth index to take into consideration the group heterogeneity by augmenting the GM index. Battese et al. (2004) employ the concept of metafrontier in capturing group heterogeneities of production activities of Indonesian garment industries by using stochastic frontier analysis (SFA). In their study, the metafrontier is equivalent to the global frontier in that the metafrontier is the envelope of the group frontiers. O'Donnell et al. (2008) also use the metafrontier to calculate technical gaps and efficiency differences of the agricultural sector among countries.

The window analysis, as discussed in Chung et al. (1997), has usually been employed in order to overcome the infeasibility problem of the cross-period DDFs of the ML index.[3] To our knowledge, attempts have not been made to overcome the infeasibility problem in the ML index other than the windows analysis.

A review of the available literature implies that several advancements have been made in the methodological development of the typical productivity growth index. To the best of our knowledge, however, this enthusiasm in methodological development has not carried over to examining the environmentally sensitive productivity growth index. Instead, previous studies mainly focus on elaboration of the ML index through studying its application. In this regard, empirical studies using the ML index have been conducted at both a micro- and macro-level. Table 1 provides the results of previous studies.

With regards to the micro-level, Chung et al. (1997) is the first one. They analyze productivity growth and its decomposed sources of Swedish paper and pulp mills for the period 1986–1990. Their empirical results suggest that technical change is the main contributor to productivity growth. Weber and Domazlicky (2001) apply the same methodology to investigate productivity growth in the US manufacturing sector for the period 1988–1994 in order to incorporate toxic release into the productivity analysis. Nakano and Managi (2008) measure productivity in the Japanese steam power-generation sector to examine the effects of industrial reforms on the productivity for the period 1978–2003. Yu et al. (2008) examine the productivity growth of Taiwan's airport sector by studying the 1995–1999 operations of four airports.

The ML index is also employed in measuring environmentally sensitive productivity growth at the macro-level. Yörük and Zaim (2005) employ both the M index and the ML index in order to analyze productivity growth and its decomposed sources in OECD countries for the period between 1985–1998. They found that Ireland and Norway were the best performers and that technical change was the main contributor to productivity growth. Färe et al. (2001) employ the ML index to account for both marketed output and the pollution abatement activities in US state manufacturing sectors from 1974 to 1986. Jeon and Sickles (2004) use the ML index to investigate different patterns in productivity growth and its decomposition between 17 OECD and 11 Asian countries over the period 1980–1990 and 1980–1995, respectively. They show that OECD countries on average grow in a lesser carbon-emitting way.[4] Methodologically, they extend the conventional ML index by introducing the bootstrapping method into the ML index. Kumar (2006) employs the ML index to analyze the environmentally sensitive productivity growth of 41 countries for the period between 1973 and 1992. In his study, Kumar found that the productivity growth of Annex-I countries are higher than that of Non-Annex-I countries, and that technical change is the main contributor to productivity growth. These studies are initiated by the fact that the most important environmental problem in recent decades is global warming, which is mainly caused by CO_2 emissions (IPCC 2007).

If employed in developing policies, the empirical results of the ML index need to be implemented with caution because it is likely to be biased due to the infeasibility and the lack of circularity. This paper demonstrates a way to overcome the problems inherent in the conventional environmentally sensitive productivity growth index. We

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[2] The GM index also does not consider the effect of environmentally harmful by-products in general. Hence, we did not include environmentally harmful by-products when using the GM index in the empirical investigation.

[3] Using a slightly different perspective, the infeasibility problem could be solved by augmenting the infeasibility problem inherent in the super-efficiency measure. The infeasibility in the super-efficiency measure is well discussed in Xue and Harker (2002) and Chen (2005). Authors are grateful to anonymous referees for their constructive comments.

[4] Jeon and Sickles (2004) show that the average M and ML indexes of OECD countries are 1.0113 and 1.0116, respectively. Those of Asian countries are 0.9996 and 0.9963, respectively.

Table 1 Recent studies on the Malmquist-Luenberger productivity growth index

Papers	Units	PC	EC	TC
Chung et al. (1997)	39 Swedish paper mills (1986–1990)	1.051	0.968	1.088
Färe et al. (2001)	48 US states manufacturing sectors (1974–1986)	1.036	1.004	1.032
Kumar (2006)	41 countries (1971–1992)	1.000	0.999	1.000
Weber and Domazlicky (2001)	48 US states (1988–1994)	1.014	–	–
Yu et al. (2008)	4 Taiwanese airport (1995–1999)	1.471	1.047	1.361
Yörük and Zaim (2005)	28 OECD countries (1985–1998)	1.095	1.028	1.065
Jeon and Sickles (2004)	17 OECD countries (1980–1990),	1.012	1.002	1.010
	11 Asian countries (1980–1995)	0.996	0.999	0.998

Rates of average efficiency change and technical change are not provided in Weber and Domazlicky (2001)

Yörük and Zaim (2005) provide cumulative indices

propose an alternative environmentally sensitive productivity growth index, called the global Malmquist-Luenberger productivity index (hereafter, GML index).

The proposed index is employed in measuring productivity growth of 26 OECD countries for the period 1990–2003. We also employed the ML, M and GM indices for comparative purposes. Through the comparison, we believe that policy makers will find practical solutions to balancing economic growth and reducing emissions of environmentally harmful by-products.

The remainder of this paper is organized as follows. A methodological discussion is provided in Sect. 2. A description of the data set and the empirical results are given in Sect. 3, followed by a brief conclusion in Sect. 4.

2 Methodology

The index we propose in this study augments the basic assumptions of the ML index. Section 2.1 provides the underlying assumptions on the production possibility set and the definition of the DDF. Then, we present our alternative GML index in Sect. 2.2.

2.1 The underlying assumptions

Under a panel of $k = 1,\ldots,K$ countries and $t = 1,\ldots,T$ time periods, the production technology for countries producing M desirable outputs, $\mathbf{y} \in R_+^M$, and J undesirable outputs, $\mathbf{b} \in R_+^J$, by using N inputs, $\mathbf{x} \in R_+^N$, is represented by the production possibility set (PPS), $\mathbf{P(x)}$. Throughout this study, we deal with desirable and undesirable outputs asymmetrically, modeling the idea that it is costly to reduce undesirable outputs. The PPS can be expressed as follows:

$$\mathbf{P(x)} = \{(\mathbf{y},\mathbf{b})|\mathbf{x}\,\text{can produce}\ (\mathbf{y},\mathbf{b})\}. \tag{1}$$

The following axioms are required for modeling the production technology:

the PPS is compact for each input and output vectors, (2a)

$$(\mathbf{0},\mathbf{0}) \in \mathbf{P(x)} \quad \text{for all}\, x \in R_+^N, \tag{2b}$$

$$\text{if}\, \mathbf{x}' \geq \mathbf{x}, \quad \text{then}\, \mathbf{P(x')} \supseteq \mathbf{P(x)}, \tag{2c}$$

$$\text{if}\ (\mathbf{y},\mathbf{b}) \in \mathbf{P(x)}\, \text{and}\, 0 \leq \theta \leq 1, \quad \text{then}\, (\theta\mathbf{y},\theta\mathbf{b}) \in \mathbf{P(x)}, \tag{2d}$$

$$\text{if}\ (\mathbf{y},\mathbf{b}) \in \mathbf{P(x)}\, \text{and}\, \mathbf{y}' \leq \mathbf{y}, \quad \text{then}\, (\mathbf{y}',\mathbf{b}) \in \mathbf{P(x)}, \tag{2e}$$

$$\text{if}\ (\mathbf{y},\mathbf{b}) \in \mathbf{P(x)}\, \text{and}\, \mathbf{b} = 0, \quad \text{then}\, \mathbf{y} = 0. \tag{2f}$$

The axiom in (2a) means that finite amounts of inputs can only produce finite amounts of outputs (Färe et al. 2007). The axiom in Eq. (2b) designates that inactivity is always possible (Färe et al. 2007). The strong disposability of inputs is assumed in the third axiom of Eq. (2c). This means that if inputs are increased (or not reduced), then the output set will not shrink (Färe et al. 2007). The axiom in Eq. (2d) designates that any proportional contraction of desirable and undesirable outputs together is feasible if the original combination of the desirable and the undesirable outputs is in the PPS. It also implies that reduction in undesirable outputs are always possible if desirable outputs are reduced in proportion, meaning that it is costly to reduce undesirable outputs (Färe et al. 2007). The strong disposability of desirable outputs is incorporated in the fifth axiom shown in Eq. (2e). In this axiom, it is assumed that if an output vector is feasible, then any output vector with less of the desirable output is also feasible. This also means that some of the desirable outputs can always be disposed of without any cost (Färe et al. 2007). The final condition in Eq. (2f) formulates the idea that if no undesirable outputs are produced, it is not possible to produce any desirable outputs. It also means that if desirable outputs are produced then some undesirable outputs must also be produced. This condition is termed null-jointness (Färe et al. 2007).

The above set representations of the technology are conceptually useful. However, they are not helpful from a computational perspective. The technology can be easily represented by means of DDFs, allowing us to maintain the above assumptions. Let $\mathbf{g} = (\mathbf{g_y}, \mathbf{g_b})$ be a direction vector, where $\mathbf{g} \in R_+^M \times R_+^J$. Then, the DDF is defined as follows:

$$\mathbf{D}(\mathbf{x}, \mathbf{y}, \mathbf{b}; \mathbf{g_y}, \mathbf{g_b}) = \max\{\beta | (\mathbf{y} + \beta \mathbf{g_y}, \mathbf{b} - \beta \mathbf{g_b}) \in \mathbf{P}(\mathbf{x})\} \tag{3}$$

This function seeks the maximal increase of desirable outputs while simultaneously reducing undesirable outputs. The direction vector, **g**, determines the direction of outputs, by which desirable outputs increase and undesirable outputs decrease. In this paper, the direction vector was taken as $\mathbf{g} = (\mathbf{y}, \mathbf{b})$ following Chung et al. (1997).

The PPS and DDF are depicted in Fig. 1. The PPS is the interior solid line and the DDF of the DMU F is represented as β. We shall omit the direction vector $\mathbf{g} = (\mathbf{y}, \mathbf{b})$ in DDFs to save space. For example, in all places we replace $\mathbf{D}(\mathbf{x}, \mathbf{y}, \mathbf{b}; \mathbf{y}, \mathbf{b})$ with $\mathbf{D}(\mathbf{x}, \mathbf{y}, \mathbf{b})$.

2.2 The ML and GML index

In order to define and decompose the GML index, two definitions of the benchmark technology are essential: contemporaneous benchmark and global benchmark technologies.

A contemporaneous benchmark technology is defined as $\mathbf{P}^t(\mathbf{x}^t) = \{(\mathbf{y}^t, \mathbf{b}^t) | \mathbf{x}^t \text{ can produce } (\mathbf{y}^t, \mathbf{b}^t)\}$, where $t = 1, \ldots, T$. The contemporaneous benchmark technology constructs a reference production set at time t. This set is made from the observations made *at that time only* (Tulkens and Vanden Eeckaut, 1995).

A global benchmark technology is defined as $\mathbf{P}^G = \mathbf{P}^1 \cup \mathbf{P}^2 \cup \cdots \cup \mathbf{P}^T$. This global benchmark technology is an augmented version of Pastor and Lovell (2005), which incorporates undesirable outputs in production activities. This global benchmark technology envelopes all contemporaneous benchmark technologies by establishing a single reference PPS from a panel data on inputs and outputs of relevant DMUs.[5]

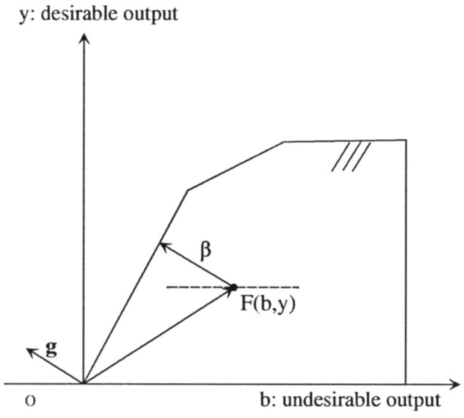

Fig. 1 Distance function and the ML index

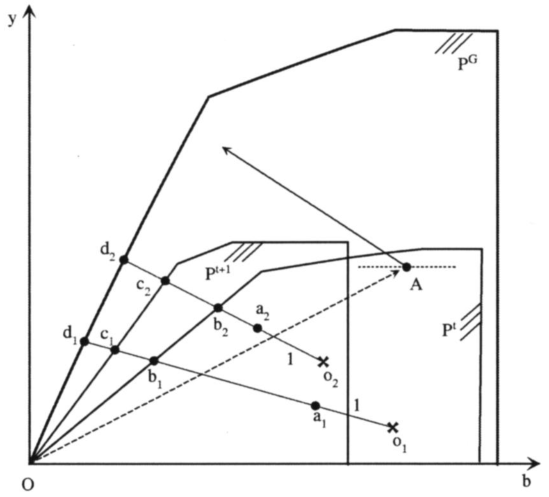

Fig. 2 Concept of the global Malmquist-Luenberger productivity index

The above two definitions of the PPSs are depicted in Fig. 2. The two interior solid lines are the contemporaneous technologies for time period t and $t+1$, respectively. The interior thick solid line is the global technology. The global benchmark technology envelopes all the contemporaneous benchmark technologies. Note that remaining $(T-2)$ contemporaneous benchmark technologies are not depicted in this Figure for simplicity. Imagine that these remaining $(T-2)$ contemporaneous benchmark technologies are depicted transparently. Then, the envelopment of all the T contemporaneous benchmark technologies is equivalent to the global benchmark technology.

As developed by Chung et al. (1997), the ML index of a country i is defined on two consecutive contemporaneous benchmark technologies, as follows:

$$\mathrm{ML}^s(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1}) = \frac{1 + \mathbf{D}^s(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t)}{1 + \mathbf{D}^s(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1})}, \tag{4}$$

where the directional distance functions $\mathbf{D}^s(\mathbf{x}, \mathbf{y}, \mathbf{b}) = \max\{\beta | (\mathbf{y} + \beta \mathbf{y}, \mathbf{b} - \beta \mathbf{b}) \in \mathbf{P}^s(\mathbf{x})\}, s = t, t+1$, are defined on the contemporaneous technology set $\mathbf{P}^s$. If a production activity enables more (less) desirable outputs and less (more) undesirable outputs, then $\mathrm{ML}^s >$ (<) 1, corresponding to productivity gain (loss). Since $\mathrm{ML}^t(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1}) \neq \mathrm{ML}^{t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1})$, the ML index is usually redefined as the geometric mean of

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[5] Berg et al. (1992) address a fixed-type benchmark technology frontier. Although the benchmark technology frontier is constructed at time period $t = 1$ or $t = T$, their work is notable in that it opens up the possibility of choosing a *fixed* benchmark technology frontier. We are grateful to Associate Editor for this invaluable comment.

the ML indices of two consecutive periods. The ML index can be decomposed into components of productivity growth, such as efficiency change and technical change, as follows:

$$
\begin{aligned}
\mathrm{ML}^{t,t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t,\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1}) &= \left[\frac{1+\mathbf{D}^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^t(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})} \times \frac{1+\mathbf{D}^{t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})}\right]^{1/2} \\
&= \frac{1+\mathbf{D}^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})} \times \left[\frac{1+\mathbf{D}^{t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)} \cdot \frac{1+\mathbf{D}^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})}{1+\mathbf{D}^t(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})}\right]^{1/2} \\
&= \frac{\mathrm{TE}^{t+1}}{\mathrm{TE}^t} \times \left[\mathrm{TG}_t^{t,t+1} \cdot \mathrm{TG}_{t+1}^{t,t+1}\right]^{1/2} \\
&= \mathrm{EC}^{t,t+1} \times \mathrm{TC}^{t,t+1},
\end{aligned}
\tag{5}
$$

where TE^s is a measure of technical efficiency at time period s; $\mathrm{TG}_s^{t,t+1}$ is a measure of technical gap between time periods t and $t+1$ along the ray from the observation at time period s in direction ($\mathbf{y}^s$, $\mathbf{b}^s$).

The efficiency change term, $\mathrm{EC}^{t,t+1}$, is a change in technical efficiency during two periods, capturing how close a DMU moves towards a contemporaneous benchmark technology at time period $t+1$ compared to time period t. $\mathrm{EC}^{t,t+1} > (<)\ 1$ corresponds to efficiency gain (loss), indicating the catching-up with (lagging behind) the contemporaneous benchmark technology frontier. The technical change term, $\mathrm{TC}^{t,t+1}$, measures a shift in a contemporaneous benchmark technology frontier. If a contemporaneous benchmark technology frontier shifts in the direction of more (less) desirable outputs and less (more) undesirable outputs, then $\mathrm{TC}^{t,t+1} > (<)\ 1$. Change in productivity is determined by the simultaneous effect of these two changes.

We provide the geometric meaning of Eq. (5) with the help of Fig. 2. Let us assume that one DMU produces outputs at a_1 and a_2 at time period t and $t+1$, respectively. Then, $\mathrm{TE}^t = 1/(1+\mathbf{D}^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)) = 1/(1+a_1b_1) = 1/o_1b_1$, where o_1 is a virtual origin of a_1. Likewise, $\mathrm{TE}^{t+1} = 1/o_2c_2$. Hence, the efficiency change, $\mathrm{EC}^{t,t+1}$, which measures how close a DMU moves towards technology frontier during two periods, is equivalent to o_1b_1/o_2c_2. $\mathrm{TG}_t^{t,t+1}$, the technical gap between time periods t and $t+1$ along the ray from the observation at time period t, is calculated as follows:

$$
\mathrm{TG}_t^{t,t+1} = \frac{1+\mathbf{D}^{t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)} = \frac{1+a_1c_1}{1+a_1b_1} = \frac{o_1c_1}{o_1b_1}. \tag{6}
$$

The technical gap measured by a_2 is calculated as $\mathrm{TG}_{t+1}^{t,t+1} = o_2c_2/o_2b_2$. Hence, $\mathrm{TC}^{t,t+1}$ measures the average of gaps between two contemporaneous technology frontiers.

It is worth to note that the cross-period DDFs, $\mathbf{D}^t(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1})$ and $\mathbf{D}^{t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t)$, are not free from the infeasibility problem in Eq. (5). The DMU A in Fig. 2 is a good example for this infeasibility problem. The $\mathbf{D}^{t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t)$ of this DMU is not feasible. In the ML index approach this kind of infeasibility problem is often resolved by employing the window analysis.

The GML index, proposed in this paper, is defined as follows:

$$
\begin{aligned}
&\mathrm{GML}^{t,t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t,\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1}) \\
&\quad = \frac{1+\mathbf{D}^G(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^G(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})},
\end{aligned}
\tag{7}
$$

where the directional distance function, $\mathbf{D}^G(\mathbf{x},\mathbf{y},\mathbf{b}) = \max\{\beta|(\mathbf{y}+\beta\mathbf{y}, \mathbf{b}-\beta\mathbf{b}) \in \mathbf{P}^G(\mathbf{x})\}$, is defined on the global technology set $\mathbf{P}^G$. If a production activity enables more (less) desirable outputs and less (more) undesirable outputs, then $\mathrm{GML}^{t,\,t+1} > (<)\ 1$, indicating productivity gain (loss).

The GML index can also be decomposed into components of productivity growth, as follows:

$$
\begin{aligned}
&\mathrm{GML}^{t,t+1}(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t,\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1}) \\
&= \frac{1+\mathbf{D}^G(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^G(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})} = \frac{1+\mathbf{D}^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t)}{1+\mathbf{D}^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1})} \\
&\times \left[\frac{(1+\mathbf{D}^G(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t))/(1+\mathbf{D}^t(\mathbf{x}^t,\mathbf{y}^t,\mathbf{b}^t))}{(1+\mathbf{D}^G(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1}))/(1+\mathbf{D}^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1}))}\right] \\
&= \frac{\mathrm{TE}^{t+1}}{\mathrm{TE}^t} \times \left[\frac{\mathrm{BPG}_{t+1}^{t,t+1}}{\mathrm{BPG}_t^{t,t+1}}\right] \\
&= \mathrm{EC}^{t,t+1} \times \mathrm{BPC}^{t,t+1},
\end{aligned}
\tag{8}
$$

where TE^s and $\mathrm{EC}^{t,t+1}$ are the same as the ML index; $\mathrm{BPG}_s^{t,\,t+1}$ is a best practice gap between a contemporaneous technology frontier and a global technology frontier, along the ray from the observation at time period s in direction ($\mathbf{y}^s$, $\mathbf{b}^s$). Hence, $\mathrm{BPC}^{t,t+1}$, which is the best practice gap change between two time periods, measures technical change between the two time periods.

We also provide the geometric meaning of Eq. (8). Since TE and EC have the same geometric meaning with the ML index, we interpret only the $\mathrm{BPG}_s^{t,t+1}, s = t, t+1$ and $\mathrm{BPC}^{t,t+1}$. The $\mathrm{BPG}_t^{t,t+1}$ in Fig. 2 is calculated as follows:

$$\mathrm{BPG}_t^{t,t+1} = 1 \Big/ \left(\frac{1 + \mathbf{D}^G(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t)}{1 + \mathbf{D}^t(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t)} \right) = 1 \Big/ \left(\frac{o_1 d_1}{o_1 b_1} \right) = \frac{o_1 b_1}{o_1 d_1}. \quad (9)$$

Hence, $\mathrm{BPG}_t^{t,t+1}$ is a proxy of the distance between the contemporaneous technology frontier and the global technology frontier along the ray from the observation at time period t in direction ($\mathbf{y}^t$, $\mathbf{b}^t$). $\mathrm{BPG}_{t+1}^{t,t+1}$ is calculated as o_2c_2/o_2d_2. Hence, $\mathrm{BPC}^{t,t+1}$, which is the ratio of two BPGs, measures how closely a contemporaneous technology frontier shifts towards the global technology frontier in the direction of more desirable outputs and less undesirable outputs. $\mathrm{BPC}^{t,t+1} > (<) 1$ corresponds to technical progress (regress).

It needs to be emphasized that the GML index circumvents the infeasibility problem because ($\mathbf{x}^s$, $\mathbf{y}^s$, $\mathbf{b}^s$) $\in \mathbf{P}^s$ and ($\mathbf{x}^s$, $\mathbf{y}^s$, $\mathbf{b}^s$) $\in \mathbf{P}^G$, $s = t, t + 1$. In this way, the GML index, unlike the ML index, is free from the infeasibility problem.

The DDFs in Eq. (5) and (8) are calculated by employing DEA-type linear programming. Issues on calculating the DDFs by means of DEA are well summarized in Kumar (2006) and Chung et al. (1997). When constructing PPSs for estimation of DDFs in this study, constant returns to scale is assumed.[6] In the interest of saving space, we omit discussion on this issue.

Proposition 2 of Appendix provides the relationship between the GML (ML) and GM (M) indices when **b** is not included in the indices. This proposition indicates that the GML (ML) index without environmentally harmful by-products is equivalent to the GM (M) index, given $\mathbf{g} = \mathbf{g}_y = \mathbf{y}$.

3 Data and empirical study

As part of the empirical study, the description on data used in this study is provided. Then the four productivity indices, i.e. GML, ML, GM and M indices, are summarized for each of the sample countries and periods. Note that the two environmentally harmful by-products are not included in calculating the GM and the M indices.

3.1 Description of the data

We obtain the data on six variables for 26 countries over the periods 1990–2003. These variables are GDP, CO_2 emissions, SO_x emissions, labor force, capital stock and commercial energy consumption. Of the first three variables, GDP is chosen as a proxy of the desirable output, and CO_2 and SO_x emissions as the proxies of the undesirable outputs. Labor force, capital stock and commercial energy consumption are chosen as inputs of production technology.

Data on GDP and labor force were collected from the Penn World Table (Mark 6.2). The capital stock information of each country was not available in the raw data set for the study period. We, therefore, estimated capital stock using capital stock data in the Penn World Table (Mark 5.6) and investment series in the Penn World Table (Mark 6.2), by employing the perpetual inventory method. We assumed a depreciation rate of 10% per year. GDP and capital stock were transformed to be measured in constant prices and in US dollars (USD) corresponding to 2000 purchasing power parities. Data on CO_2 emissions per capita and energy consumption per capita were taken from the World Development Indicators website. These were multiplied by the populations of each country in order to calculate total CO_2 emissions and total energy consumption on a nation-wide level. Data on SO_x emissions were retrieved from the Key Environmental Indicators for each year.

Descriptive statistics of variables used in this study are presented in Table 2. In all variables, the mean value is noticeably larger than the median value. This suggests that the distributions of the variables are skewed to the right, indicating that most countries are observed near the left tail of the distribution.

Average level and average growth rate of the variables for each country are listed in Table 3. The average level of GDP of our sample is 875.1 billion USD. The United States (8493.6 billion USD) had the highest average level of GDP, followed by Japan (2916.9 billion USD) and Germany (1918.7 billion USD). The average annual growth

Table 2 Descriptive statistics of input/output variables used in this study

	Mean	SD	Median	Max	Min
GDP (Bil. USD)	875.1	1683.3	267.7	10205.1	5.4
CO_2 (Mil. Metric Tons)	446.1	1041.1	122.9	5959.8	1.8
Energy (MTOE)	185.3	409.1	57.8	2306.6	2.1
SO_x (Tons)	1432.9	3319.4	420.5	20924.0	2.0
Labor (Millions)	19.4	28.9	6.1	150.4	0.1
Capital (Bil. USD)	1656.0	2968.2	499.8	17701.9	10.7

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[6] When variable returns to scale is assumed, it is needed to check if the non-convexity problem occurs (Kuosmanen 2005; Kuosmanen and Podinovski 2009). An empirical investigation shows that the CRS (VRS) assumption yields a convex (non-convex) PPS. We are grateful to Associate Editor for this comment.

Table 3 Growth rates of input/output variables used in this study

Country	GDP (Bil. USD)		CO_2 (Mil. Metric Tons)		SO_x (Tons)		Energy (MTOE)		Labor (Millions)		Capital (Bil. USD)	
	Level	Growth	Level	Growth	Level	Growth	Level	Growth	Level	Growth	Level	Growth
Australia	437.0	3.59	313.8	0.95	2016.6	4.34	100.4	1.97	9.3	1.38	844.6	3.42
Austria	198.8	2.07	60.8	1.63	45.1	−6.45	28.3	2.11	3.7	0.43	401.1	2.31
Belgium	232.1	1.86	102.5	0.12	215.4	−6.85	55.1	1.46	4.2	0.43	442.4	2.30
Canada	714.2	2.91	512.0	3.26	2523.4	−2.65	234.6	1.73	15.9	1.14	1407.5	2.82
Denmark	133.4	2.15	54.6	0.70	110.2	−13.20	20.0	1.15	2.9	0.06	251.9	2.11
Finland	103.2	1.73	56.3	2.21	123.6	−6.94	32.0	1.94	2.6	0.10	240.8	−0.13
France	1350.6	1.87	363.0	0.14	900.3	−7.40	249.7	1.34	26.1	0.72	2657.4	1.90
Germany	1918.7	1.60	842.8	−1.50	1867.9	−17.36	346.8	−0.20	40.4	0.14	3967.6	1.38
Greece	140.9	2.77	82.0	2.17	521.1	1.23	25.3	2.29	4.6	1.14	270.4	1.82
Iceland	6.3	2.46	2.0	0.54	7.9	0.00	2.7	3.42	0.2	1.16	11.9	2.12
Ireland	73.3	6.71	36.8	2.44	150.9	−6.56	12.5	2.84	1.5	2.05	105.7	5.16
Italy	1212.5	1.40	412.1	1.09	1148.4	−9.45	162.3	1.54	25.1	0.34	2345.3	1.27
Japan	2916.9	1.06	1150.6	1.07	938.8	−0.86	496.5	1.14	66.9	0.49	7721.8	2.00
Korea, Republic of	622.9	5.45	370.6	4.87	664.9	−4.32	155.9	6.14	22.4	1.77	1451.3	7.14
Luxembourg	16.9	4.71	9.2	0.01	7.6	−14.97	3.7	1.36	0.2	1.14	31.3	4.77
Mexico	703.4	2.75	419.0	0.39	2726.1	−0.24	141.5	1.93	37.7	2.74	1022.6	3.20
Netherlands	369.9	2.29	144.9	0.11	117.6	−8.45	73.8	1.49	7.2	0.58	687.0	2.05
New Zealand	71.0	3.10	28.2	2.74	60.9	3.12	16.3	1.69	1.8	1.40	128.7	2.39
Norway	131.2	3.27	44.1	6.97	32.7	−6.27	24.5	1.79	2.3	0.81	285.6	1.59
Portugal	158.2	2.16	51.8	2.27	307.6	−3.54	22.0	2.87	5.1	0.63	279.2	3.79
Spain	709.3	2.79	249.7	2.88	1739.9	−4.12	109.8	3.09	17.2	1.22	1359.3	3.21
Sweden	201.9	1.79	49.5	0.46	66.5	−6.87	50.0	0.55	4.8	0.41	369.3	1.03
Switzerland	194.8	0.80	41.1	−0.44	26.1	−6.96	25.9	0.56	3.7	0.45	505.2	0.88
Turkey	340.5	2.96	183.0	3.16	1698.5	1.62	65.6	3.05	29.4	2.42	416.3	5.09
UK	1300.4	2.42	580.2	0.01	2110.4	−10.36	226.3	0.70	29.0	0.41	1976.4	2.58
USA	8493.6	2.96	5438.0	1.61	17126.9	−3.49	2136.1	1.30	139.6	1.17	13874.8	3.67
Total	875.1	2.68	446.1	1.53	1432.9	−5.27	185.3	1.89	19.4	0.95	1656.0	2.69

rate in GDP is 2.7% for our sample. Ireland had the highest average annual growth rate in GDP (6.7%), followed by Korea (5.5%) and Luxembourg (4.7%).

The average level of CO_2 emissions of our sample countries is 446.1 million metric tons. The United States (5,438.0 million metric tons) had the highest average level of CO_2 emissions, followed by Japan (1,150.6 million metric tons) and Germany (842.8 million metric tons). The average annual growth rate in CO_2 emissions is 1.5% for our sample. Norway had the highest annual growth rate (7.0%), followed by Korea (4.9%) and Canada (3.3%). Germany (−1.5%) and Switzerland (−0.4%) showed negative growth rates in CO_2 emissions.

The average level of SO_x emissions of our sample is 1.4 million kilograms. The United States (17.1 million kilograms) had the highest average level of SO_x emissions, followed by Mexico (2.7 million kilograms) and Canada (2.5 million kilograms). The average annual growth rate in SO_x emissions is −5.3% for our sample. Most sample countries showed negative growth rates of SO_x emissions. Among the sample countries, Germany showed the most dramatic decrease in SO_x emissions (−17.4%), followed by Luxembourg (−15.0%) and Denmark (−13.2%).

The average annual growth rate of energy use is 1.9 %. Among our sample countries, Korea (6.1%) showed the highest annual growth rate in energy use, followed by Iceland (3.4%). Germany (−0.2%) showed the negative growth rate in energy use. The United States (2,136.1 MTOE) and Japan (496.5 MTOE) showed the highest level of energy use among our sample. The average labor force is 19.4 million and its growth rate is 1.0%. Mexico (2.7%) and Turkey (2.4%) showed the highest growth rates in labor force. Average capital stock of our sample countries is 1,656 billion USD and its average annual growth rate is 2.7%. Korea (7.1%) and Ireland (5.2%) showed the highest growth rates in capital stock.

3.2 Comparison of the productivity indices

Before presenting temporal patterns and country-specific measures of productivity growth and its decomposed components, it is worthwhile to compare measures among indices at the non-aggregated level.[7] For the purpose of this comparison, we used two tests. First, as common in efficiency and productivity analysis studies, we employed the Wilcoxon test to test the null that measures are the same between indices. Second, we examined kernel density plots and tested the null that density plots are similar everywhere by employing the goodness-of-fit test of Fan and Ullah (1999) (hereafter, Fan-Ullah test).[8] We employed the above two tests due to the fact that similar distributions of productivity growth may yield different ranks across indices, or *vice versa*. These two tests yield robust results for testing the null. The results of the Wilcoxon test are listed in Table 4. Kernel density plots are depicted in Fig. 3, and the results of the Fan-Ullah tests are listed in Table 5. Our main focus is the comparison between the GML and the GM indices. Note again that the GM index is equivalent to the GML index without environmentally harmful by-products.

We begin with a comparison of the productivity growth measures. As can be seen in Table 4a, we reject the null that the GML and GM have the same ranks at the 1% level of significance. This signifies that two productivity growth indices yield very different ranks, indicating that the rank of a DMU changes when environmentally harmful by-products are included in the productivity analysis. Different patterns between the two measures can also be found in the kernel density plots in Fig. 3a. The test statistics of the Fan-Ullah test in Table 5a also signifies that these two kernel densities are not similar everywhere.

The comparison results of efficiency change indices are presented in Table 4b, Fig. 3b and Table 5b. It can be found that the efficiency change index of the GML is different from that of the GM index. The results of the test for the rate of technical change indices are provided in Table 4c, Fig. 3c and Table 5c. The results show that the rates of technical change between the GML and GM indices are significantly different. From these results, we reject the null that two decomposed components of the GML and the GM are the same. This difference mainly comes from the fact that the environmentally harmful by-products are included in the GML index, resulting from negative externalities.

Table 4 Results of the Wilcoxon test for productivity growth indices, rates of efficiency change and rates of technical change (H_0: Two measures have the same ranks)

	ML	GM	M
(a) Productivity growth			
GML	58,190 (0.67)	46,626[a] (0.00)	25,358[a] (0.00)
ML		45,375[a] (0.00)	23,202[a] (0.00)
GM			34,439[a] (0.00)
(b) Efficiency change			
GML	57,122 (1.00)	65,932[a] (0.00)	65,892[a] (0.00)
ML		65,932[a] (0.00)	65,892[a] (0.00)
GM			57,093 (0.99)
(c) Rate of technical change			
GML	59,355 (0.38)	46,183[a] (0.00)	23,624[a] (0.00)
ML		44,552[a] (0.00)	20,586[a] (0.00)
GM			38,713[a] (0.00)

GML, ML, GM and M are productivity growth measures of this study, Chung et al. (1997), Pastor and Lovell (2005) and Färe et al. (1994), respectively

Numbers without parentheses are test statistics

Numbers in parentheses are *p*-values

[a] Significant level at 1%

We also tested the null that the GML is the same with the conventional ML index. The results in Tables 4 and 5 provide conclusive evidence that there are differences between the two indices. That is, although two measures yield similar ranks, their density functions are significantly different. This comes from the fact that the shapes of the constructed PPSs differ considerably between them. The technical change components between these indices also differ significantly.

In summary, not only the GML and GM indices but also their decomposed sources are very different. The GML is also significantly different from the typical ML index. For comparison purposes, we provide the results obtained in the literature carrying out similar exercises.[9] By employing the paired *t*-test Kumar (2006) shows that the components of the ML index is the same as those of the M index, whereas the ML index is different from the M index. Chung et al. (1997) report similar results with ours, rejecting the null that the ML index is the same as the M index by employing various non-parametric tests.

3.3 Temporal trends of the GML and GM indices

The approach described in the Methodology section constructs the best-practice global technology frontier from the data. First, we report the annual cumulative growth of productivity and its decomposed components. The

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[7] In calculating the ML index, a linear programming infeasibility occurred. When this problem occurred, we employed the window analysis.

[8] Authors are grateful to anonymous referees for their constructive comments on this nonparametric test.

[9] Note that previous literature employ the ML and M indices.

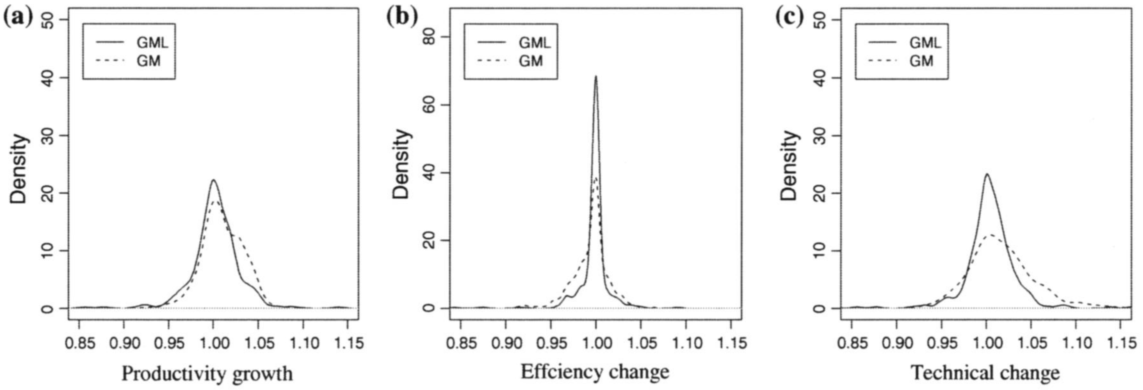


Fig. 3 Kernel density plots of productivity growth, efficiency change and technical change of GML and GM indices

Table 5 Results of goodness-of-fit test (Fan and Ullah 1999) for productivity growth indices, rates of efficiency and technical change (H_0: Two measures have the same kernel densities)

	ML	GM	M
(a) Productivity growth			
GML	107.3^a (26.04)	29.83^c (21.45)	238.88^a (26.72)
ML		173.24^a (28.58)	481.70^a (27.86)
GM			110.70^a (35.29)
(b) Efficiency change			
GML	0.00 (28.41)	325.89^a (47.27)	325.89^a (30.85)
ML		325.89^a (47.27)	325.89^a (30.85)
GM			0.00 (49.52)
(c) Rate of technical change			
GML	44.46^a (22.60)	43.44^a (18.16)	126.41^a (24.20)
ML		138.38^a (22.47)	247.73^a (29.29)
GM			38.77^b (30.97)

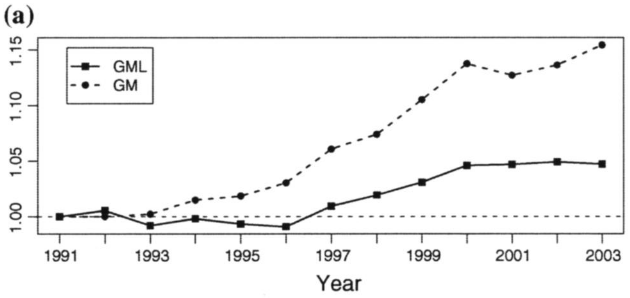
GML, ML, GM and M are productivity growth measures of this study, Chung et al. (1997), Pastor and Lovell (2005) and Färe et al. (1994), respectively

Numbers without parentheses are test statistics

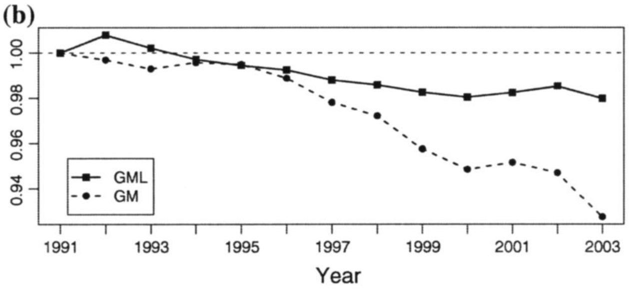
^{a}, ^{b} and ^{c} significant levels at <1%, 1–5% and 5–10%, respectively. Numbers in parentheses represent the critical values at the relevant significant levels

measures were calculated as the sequential multiplicative sums of the average annual index of productivity growth and its components, respectively. We calculated the measures for each of the GML and GM indices. Since the measures were calculated based on the *average* annual rate of change in the relevant indices, the trends can be considered as those of an *average* country in our sample. The temporal trends of cumulative productivity growth, efficiency change and technical change are depicted in each panel of Fig. 4.

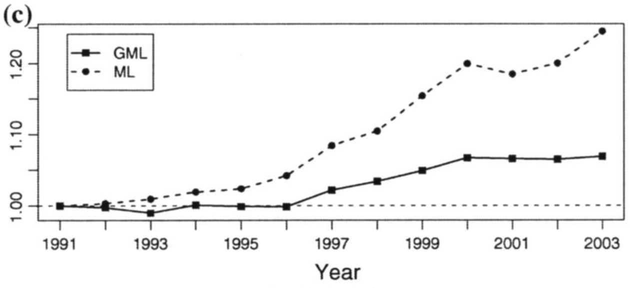
The first impression in Fig. 4a is that the productivity of the average country has been accumulated for the study period. However, the cumulative productivity level of the GM is larger than that of the GML index at the end of the studied period by around 10%p. This means that the exclusion of environmentally harmful by-products overestimates the rate of productivity growth. This is in line with the discussion that the productivity measures calculated by the two methodologies are different, as discussed in Sect. 3.2. The rate of cumulative productivity growth measured



Productivity growth



Effciency change



Technical change

Fig. 4 Cumulative productivity growth, efficiency change and technical change: GML and ML indices by the GML index is 4.7%.[10] It needs to compare our results with those obtained in the previous studies. Kumar (2006) reports that the average ML index is almost the same as the average M index when CO_2 emissions is incorporated in the ML index. Yörük and Zaim (2005) report that the ML index is larger than the M index for the period between 1985 and 1998 when incorporating CO_2, NO_x and organic water pollutant emissions into the ML index. As Yörük and Zaim (2005) argue, the ML index is smaller (larger) than the M index during time periods exhibiting a upward (downward) trend in undesirable outputs. We have already found that CO_2 emissions increased and SO_x emissions decreased for the study period, as discussed in Sect. 3.1. By integrating our descriptive statistics for the emissions of the undesirable outputs with the argument of Yörük and Zaim (2005), in our study it can be concluded that the increase of CO_2 emissions dominates the decrease of SO_x emissions in determining the relationship between the GML (ML) and GM (M) indices.

The cumulative efficiency change indices of the GML and GM indices are shown in Fig. 4b. Although both of the two cumulative efficiency change indices show decreasing patterns during the studied period, the levels at the end of the study period are different. The cumulative efficiency change index of the GML (GM) is 0.980 (0.928), indicating efficiency deterioration of 2.0% (7.2%).[11] Efficiency deterioration of the GML index occurs under the two following conditions: (a) when frontier technology advances, the technical catching-up speed of the average country is slower than that of frontier technology advancement; or (b) when frontier technology deteriorates, the technical deterioration speed of the average country is faster than that of frontier technology deterioration. Therefore, efficiency deterioration needs to be investigated along with a trend of technical change. This will be discussed later.

The cumulative technical change is depicted in Fig. 4c. Although the levels of cumulative technical change measured by the two methodologies show increasing trends, they are different at the end of the study period. The cumulative technical change of the GM index is much larger than that of the GML index by around 18%p. This difference appears to come from the exclusion of environmentally harmful by-products in the GM index. Compared with the base year (1990), the technology level of the average country can be regarded as having progressed by approximately 6.9% from the enviro-economic perspective.[12]

Now let us investigate the relationship between efficiency *deterioration* and technology *progress*. As discussed in the Methodology section, the technical progress corresponds to a shift in the technology frontier in the direction of more GDP and fewer CO_2 and SO_x emissions. Combining efficiency deterioration with this technical progress, it can be concluded that the average country has lagged behind the technology frontier, as in the first possibility discussed above.

Regardless of the methodology employed, productivity growth almost coincides with technical change, rather than efficiency change. Hence, productivity growth is mainly attributed to the technical rather than the efficiency change. This finding corresponds to the results of Jeon and Sickles (2004), Kumar (2006) and Yörük and Zaim (2005). The result implies that in fostering productivity it is more effective to invent environment-related technologies rather than to catch up with the world frontier technology.

3.4 Country heterogeneity

Average productivity growth, efficiency change and technical change indices are calculated for the sample countries. In order to compare the results among indices, we also calculated corresponding measures for the ML, GM and M indices. The results are listed in Table 6. Here the average is calculated by means of weighted geometric means, where the weight is the GDP. Recall that index values greater (less) than unity indicate improvement (deterioration) in the relevant performance.

As indicated in the last row of Table 6, *overall* environmentally sensitive productivity growth indices are less than the typical productivity growth indices. For example, the GML index yields annual productivity growth of 0.24%, while the GM index is 1.02%.[13] It appears that the environmentally sensitive productivity growth indices are less than the typical productivity growth indices because negative externalities of CO_2 and SO_x emissions are accounted for in the former. In this sense, the typical productivity growth indices are not suitable for measuring productivity in the presence of negative externalities.

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[10] This rate of productivity growth is less than that of Zhou et al. (2010), of which cumulative productivity growth rate is 24.6%. The difference between this study and Zhou et al. (2010) comes from (a) different sample countries, (b) different study period and (c) inputs/outputs selection.

[11] Cumulative efficiency change in Zhou et al. (2010) is minus 4.5%. See footnote 10 for reasons of the difference.

[12] Zhou et al. (2010) report that cumulative technical change is 30.4%, which is significantly larger than our result. For the reasons of the difference, see footnote 10

[13] As discussed in footnote 4, Jeon and Sickles (2004) show that for OECD countries the ML index is slightly larger than the M index. The difference between Jeon and Sickles (2004) and this study seems to come from (a) the different time span, and (b) the different sample countries including the rapid carbon-emitting countries such as Korea and Turkey.

Table 6 Productivity growth, efficiency change and technical change of OECE countries (1990–2003): GML (this study), ML (Chung et al., 1997), GM (Pastor and Lovell, 2005) and M (Färe et al. 1994) indices

Country	GML			ML			GM			M		
	PC	EC	BPC	PC	EC	TC	PC	EC	BPC	PC	EC	TC
Australia	1.0210	1.0000	1.0210	1.0061	1.0000	1.0061	1.0173	0.9904	1.0272	1.0369	0.9904	1.0470
Austria	1.0031	1.0000	1.0031	0.9983	1.0000	0.9983	1.0094	0.9914	1.0182	1.0207	0.9914	1.0296
Belgium	1.0041	0.9939	1.0102	1.0047	0.9939	1.0108	1.0167	0.9859	1.0312	1.0190	0.9859	1.0335
Canada	0.9940	0.9821	1.0121	0.9903	0.9821	1.0084	1.0122	0.9942	1.0181	1.0307	0.9942	1.0367
Denmark	1.0129	1.0000	1.0129	1.0011	1.0000	1.0011	1.0092	1.0015	1.0077	1.0216	1.0015	1.0201
Finland	1.0028	0.9974	1.0054	1.0083	0.9974	1.0109	1.0196	0.9996	1.0199	1.0198	0.9996	1.0202
France	1.0044	1.0000	1.0044	1.0049	1.0000	1.0049	1.0228	0.9856	1.0378	1.0192	0.9856	1.0341
Germany	1.0077	1.0008	1.0069	1.0063	1.0008	1.0055	1.0223	0.9918	1.0307	1.0158	0.9918	1.0242
Greece	1.0061	1.0139	0.9923	1.0139	1.0139	1.0000	1.0112	0.9950	1.0163	1.0295	0.9950	1.0347
Iceland	1.0081	1.0044	1.0037	1.0093	1.0044	1.0048	1.0103	0.9939	1.0165	1.0263	0.9939	1.0325
Ireland	1.0089	1.0051	1.0038	1.0023	1.0051	0.9972	1.0097	1.0062	1.0035	1.0725	1.0062	1.0659
Italy	0.9968	0.9985	0.9983	0.9972	0.9985	0.9988	1.0020	0.9970	1.0050	1.0142	0.9970	1.0172
Japan	0.9973	0.9849	1.0126	1.0091	0.9849	1.0245	1.0069	0.9924	1.0146	1.0106	0.9924	1.0184
Korea, Republic of	0.9977	0.9875	1.0103	0.9968	0.9875	1.0094	1.0114	0.9789	1.0332	1.0549	0.9789	1.0776
Luxembourg	1.0046	1.0000	1.0046	1.0003	1.0000	1.0003	1.0125	1.0000	1.0125	1.0481	1.0000	1.0481
Mexico	0.9912	0.9946	0.9965	0.9978	0.9946	1.0031	0.9991	0.9977	1.0014	1.0281	0.9977	1.0305
Netherlands	1.0126	0.9993	1.0133	1.0090	0.9993	1.0096	1.0206	0.9870	1.0340	1.0230	0.9870	1.0364
New Zealand	1.0054	0.9966	1.0088	1.0103	0.9966	1.0137	1.0116	0.9958	1.0159	1.0322	0.9958	1.0365
Norway	1.0118	1.0052	1.0066	1.0107	1.0052	1.0055	1.0195	0.9986	1.0209	1.0322	0.9986	1.0336
Portugal	0.9959	1.0000	0.9959	0.9745	1.0000	0.9745	1.0004	0.9927	1.0078	1.0218	0.9927	1.0293
Spain	0.9944	0.9983	0.9961	0.9863	0.9983	0.9880	1.0090	0.9887	1.0205	1.0293	0.9887	1.0411
Sweden	1.0074	1.0000	1.0074	1.0116	1.0000	1.0116	1.0128	0.9957	1.0172	1.0190	0.9957	1.0234
Switzerland	1.0036	1.0000	1.0036	1.0144	1.0000	1.0144	1.0026	1.0000	1.0026	1.0082	1.0000	1.0082
Total	1.0024	0.9985	1.0039	1.0026	0.9985	1.0041	1.0102	0.9943	1.0160	1.0271	0.9943	1.0331
Turkey	0.9925	1.0000	0.9925	1.0001	1.0000	1.0001	0.9896	1.0001	0.9895	1.0306	1.0001	1.0305
U.K.	0.9978	0.9966	1.0012	1.0112	0.9966	1.0147	1.0102	0.9912	1.0192	1.0251	0.9912	1.0342
U.S.A.	0.9916	1.0000	0.9916	0.9968	1.0000	0.9968	1.0115	0.9895	1.0222	1.0301	0.9895	1.0411
Average	1.0024	0.9985	1.0039	1.0026	0.9985	1.0041	1.0102	0.9943	1.0160	1.0271	0.9943	1.0331

The examination of country-specific productivity growth provides a somewhat different relationship between the GML (ML) and the GM (M) indices. For example, Switzerland shows average annual growth of 0.36% by the GML index and 0.26% by the GM index. For some countries the gap between the environmentally sensitive productivity growth index and typical productivity index is negligible, while for other countries it is not.

If a country has the environmentally sensitive productivity growth index larger than the typical productivity growth index (for example, GML vs. GM), the country is considered to have harmonized economic growth with the decrease of CO_2 and SO_x emissions. If a country's environmentally sensitive productivity growth index is smaller than the typical productivity growth index, the decrease of CO_2 and SO_x emissions is less emphasized than the increase of GDP. By this criterion of *eco-friendliness*, countries are categorized into two groups. In this study, we named the countries in the former group ***green countries***, and the countries in the latter group ***yellow countries***. Since we have the two environmentally sensitive productivity growth indices (GML and ML) and the two typical productivity indices (GM and M), we used GML/GM and ML/M as the pairings for this categorization. The results are listed in Table 7. The first column lists the green and yellow countries categorized by the GML/GM criterion, and the second column lists those countries by the ML/M criterion. The third column lists the countries shown in both of the first and second columns. Switzerland is found to be the green country regardless of the criteria, while Canada, Italy, Korea, Portugal, Spain and the United States are found to be the yellow countries in both of the criteria. This result coincides with the discussion in Sect. 3.1. For example, Switzerland has a positive GDP growth rate and

Table 7 Green countries and yellow countries

	(1) GML and GM	(2) ML and M	(3) Intersect
Green countries	Australia, Denmark, Switzerland	Switzerland	Switzerland
Yellow countries	Canada, Italy, Japan, Korea, Portugal, Spain, UK, USA	Austria, Canada, Italy, Korea, Mexico, Portugal, Spain, USA	Canada, Italy, Korea, Portugal, Spain, USA

Intersect in the last column represents Green (or, yellow) countries listed in both column (1) and column (2)

negative growth rate in CO_2 and SO_x emissions, and the United States has the highest level of CO_2 and SO_x emissions.[14]

3.5 Innovative countries

The technical change index for any one particular country in two consecutive years, if not on the frontier, is not necessarily an index of the shift in the world technology frontier. Hence, a value of this index greater than unity does not necessarily imply that the country under consideration actually pushes the world technology frontier outwards in the direction of more desirable outputs and less undesirable outputs. Additional information is necessary in order to determine which countries are the world innovators. The following three conditions help us determine this issue:

$$\mathrm{BPC}^{t,t+1} > 1 \tag{10a}$$

$$\mathbf{D}^{t}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1}) < 0 \tag{10b}$$

$$\mathbf{D}^{t+1}(\mathbf{x}^{t+1},\mathbf{y}^{t+1},\mathbf{b}^{t+1}) = 0. \tag{10c}$$

As discussed earlier, the first condition indicates that the world technology frontier is shifted in the direction of more desirable outputs and fewer undesirable outputs. This means that in period $t + 1$ it is possible to increase GDP and to decrease the level of CO_2 and SO_x emissions relative to period t. This measures the shift in the relative portions of the frontier between period t and $t + 1$ for a given country. The second condition indicates that production in period $t + 1$ occurs outside the production possibility set of period t. This implies that technology of period t is not possible to produce the output vector of period $t + 1$ with the input vector of period $t + 1$. Hence, the value of the directional distance function evaluating input/output vector at period $t + 1$ relative to the reference technology of period t is less than zero. The third condition indicates that the country should be on the world technology frontier in period $t + 1$. It should be noted that, since our sample countries contain all advanced countries, we are confident that the estimated frontier represents the world frontier technology.

Table 8 lists the innovative countries for each year, captured by the GML index. Out of 26 OECD countries, thirteen countries are recorded as innovative.[15] Those countries are Austria, Denmark, France, Iceland, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the UK and the USA. Some countries are innovative only for a short period, e. g. Denmark, the UK and the USA. While other, like Sweden, is a innovator for almost the entire study period. As expected, our result supports that low CO_2 and SO_x emitters coupled with high GDP are innovative countries. For example, high CO_2 emitting countries, such as Ireland and Korea, are not found to be innovators despite the fact that GDP growth rates are quite high.

We also list innovative countries captured by the ML index in the last column of Table 8. Interestingly, fifteen countries registered as innovative by the ML index, a 15% increase from our GML index results. In this sense, it appears that the GML index has a higher discriminating power than the ML index in identifying innovators. Annual investigation also reveals similar results for almost the entire study period. For example, in 1994–1995, only two countries are registered as innovative countries by the GML index, whereas four countries by the ML index.

4 Conclusion

Even though the geometric mean form of the Malmquist-Luenberger (ML) index has long been used in measuring environmentally sensitive productivity growth, one of its drawbacks is that it is not circular. The conventional ML

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[14] The results of Jeon and Sickles (2004) reveal that six countries (China, France, Italy, Japan, Spain and Sweden) are green countries. Their results also reveal that 10 countries (Austria, Belgium, Canada, Finland, Germany, Ireland, Japan, Norway, Korea and Taiwan) are yellow countries. Seven out of remaining 11 countries cannot be compared because their indexes are not available.

[15] Kumar (2006) reports that six countries (Hong Kong, Iceland, Japan, Luxembourg, the Netherlands and Switzerland) are innovators during the period between 1971–1992, which is smaller than ours. It appears that increase in the number of innovators during our study period (1990–2003) reflects the fact that the advancement of environment-related technologies has been made.

Table 8 Innovative countries

Year	GML index	ML index
1990–1991	Netherlands	Mexico, Netherlands, Spain
1991–1992	France, USA	France, USA
1992–1993	New Zealand, Sweden	New Zealand, Portugal, Sweden
1993–1994	France, New Zealand, Norway, Portugal, Switzerland	France, New Zealand, Norway, Sweden, Switzerland
1994–1995	Denmark, Sweden	Denmark, Italy, Norway, Sweden
1995–1996	Portugal, Spain	Spain
1996–1997	France, Sweden, UK	France, Sweden, UK
1997–1998	Norway, Spain, Sweden	Norway, Spain, Sweden
1998–1999	France, Sweden, Switzerland	France, Sweden, Switzerland
1999–2000	France, Portugal, Sweden, Switzerland	France, Sweden, Switzerland
2000–2001	Iceland, Portugal, Spain	Iceland, Portugal, Spain
2001–2002	Austria, France, Switzerland	Austria, France, Switzerland
2002–2003	Portugal, Sweden, Switzerland	Sweden, Switzerland

index also has a possibility of having an infeasibility problem. In order to overcome these weaknesses of the ML index, we proposed an alternative index in this paper, called the global Malmquist-Luenberger (GML) index.

The GML index is easy to calculate. Moreover, it is free from choosing benchmark technologies since a single global PPS is used as a benchmark technology for all DMUs over the whole periods. However, it is imperative to recalculate the GML index if time periods are added to a data set. It might be arguable that the recalculation is a disadvantage of the GML index. Addition of new time periods to the existing data set corresponds to the inclusion of new information to our understanding of the past. Hence, the inclusion of the new time periods allows us to understand the past in more informative perspective. In this sense, the recalculation process is not a disadvantage but an advantage.

The index is employed in measuring the environmentally sensitive productivity growth of 26 OECD countries for the period of 1990–2003. The results show that (a) the GML and the GM indices yield very different productivity growth measures and decompositions, (b) productivity growth is mainly attributed to technical change, (c) Switzerland is a green country, while Canada, Italy, Korea, Portugal, Spain and the United States are yellow countries, and (d) Sweden is found to be innovative countries for almost the entire study period.

Overall, the typical productivity growth indices (the GM and M indices) measure higher productivity growth than the environmentally sensitivity productivity growth indices (the GML and ML indices). This comes from the fact that the typical productivity growth index do not take into account negative externalities of CO_2 and SO_x emissions. Especially during the period of increasing CO_2 emissions, the importance of the environmentally sensitivity productivity growth index should be emphasized to suitably account for negative externalities of emissions. Hence, appropriate augmentation of existing environmentally sensitive productivity growth is needed. To this end, this paper contributes to fill the gap between the environmentally sensitivity productivity growth and the circularity and infeasibility problem.

We believe this paper paves the way for further methodological investigation of the ML index and the circularity condition. A possible extension of the index could be the integration of the DMU-specific time neutrality into the ML index. In doing so, it is needed to reinterpret the theoretical proposition of Pastor and Lovell (2007) from the view point of the directional distance function. With this reinterpretation, the relationship between the GML index and the firm-specific time neutral ML index could be investigated.

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Appendix

Proposition 1 *The GML index and its decomposed components are circular.*

Proof In order to save space, we substitute input and output vector in a parenthesis of a distance function with a time index. For example, $\mathbf{D}^t(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t)$ is equivalent to $\mathbf{D}^t(t)$. Then,

$$\begin{aligned}
&\mathrm{GML}^{t,t+1} \times \mathrm{GML}^{t+1,t+2} \\
&= \frac{1 + \mathbf{D}^G(t)}{1 + \mathbf{D}^G(t+1)} \times \frac{1 + \mathbf{D}^G(t+1)}{1 + \mathbf{D}^G(t+2)} \\
&= \frac{1 + \mathbf{D}^G(t)}{1 + \mathbf{D}^G(t+2)} = \mathrm{GML}^{t,t+2}. \\
&\mathrm{EC}^{t,t+1} \times \mathrm{EC}^{t+1,t+2} \\
&= \frac{1 + \mathbf{D}^t(t)}{1 + \mathbf{D}^{t+1}(t+1)} \times \frac{1 + \mathbf{D}^{t+1}(t+1)}{1 + \mathbf{D}^{t+2}(t+2)} \\
&= \frac{1 + \mathbf{D}^t(t)}{1 + \mathbf{D}^{t+2}(t+2)} = \mathrm{EC}^{t,t+2}. \\
&\mathrm{BPC}^{t,t+1} \times \mathrm{BPC}^{t+1,t+2} \\
&= \frac{\frac{1+\mathbf{D}^G(t)}{1+\mathbf{D}^t(t)}}{\frac{1+\mathbf{D}^G(t+1)}{1+\mathbf{D}^{t+1}(t+1)}} \times \frac{\frac{1+\mathbf{D}^G(t+1)}{1+\mathbf{D}^{t+1}(t+1)}}{\frac{1+\mathbf{D}^G(t+2)}{1+\mathbf{D}^{t+2}(t+2)}} = \frac{\frac{1+\mathbf{D}^G(t)}{1+\mathbf{D}^t(t)}}{\frac{1+\mathbf{D}^G(t+2)}{1+\mathbf{D}^{t+2}(t+2)}} = \mathrm{BPC}^{t,t+2}.
\end{aligned}$$

□

Proposition 2 *If undesirable outputs are excluded from directional distance functions and* $\mathbf{g} = \mathbf{g_y} = \mathbf{y}$, *then* GML ≡ GM *and* ML ≡ M.

Proof PPS is redefined as follows.

$$\Psi(\mathbf{x}) = \{\mathbf{y} \mid \mathbf{x}\,\text{can produce}\,\mathbf{y}\} \tag{11}$$

Then

$$\mathbf{D}(\mathbf{x}, \mathbf{y}; \mathbf{y}) = \max\{\beta \mid (\mathbf{x}, \mathbf{y} + \beta\mathbf{y}) \in \Psi(\mathbf{x})\}. \tag{12}$$

Then $\mathbf{D}(\mathbf{x}, \mathbf{y} \mid \mathbf{y}) = 1/D(\mathbf{x}, \mathbf{y}) - 1$, where $D(\mathbf{x}, \mathbf{y}) = \min\{\delta \mid \mathbf{y}/\delta \in \Psi(\mathbf{x})\}$ is an output-oriented distance function. This relationship can be shown as follows:

$$\begin{aligned}
\mathbf{D}(\mathbf{x}, \mathbf{y} \mid \mathbf{y}) &= \max\{\beta \mid (\mathbf{y} + \beta\mathbf{y}) \in \Psi(\mathbf{x})\} \\
&= \max\{\beta \mid D(\mathbf{x}, (1+\beta)\mathbf{y}) \le 1\} \\
&= \max\{\beta \mid (1+\beta) D(\mathbf{x}, \mathbf{y}) \le 1\} \\
&= \max\left\{\beta \,\middle|\, \beta \le \frac{1}{D(\mathbf{x}, \mathbf{y})} - 1\right\} \\
&= \frac{1}{D(\mathbf{x}, \mathbf{y})} - 1.
\end{aligned} \tag{13}$$

Then GML ≡ GM since

$$\begin{aligned}
\mathrm{GML}^{t,t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}) &= \frac{1 + \mathbf{D}^G(\mathbf{x}^t, \mathbf{y}^t, \mathbf{b}^t)}{1 + \mathbf{D}^G(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}, \mathbf{b}^{t+1})} \\
&= \frac{1 + \left(-1 + \frac{1}{D^G(\mathbf{x}^t, \mathbf{y}^t)}\right)}{1 + \left(-1 + \frac{1}{D^G(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}\right)} \\
&= \frac{D^G(\mathbf{x}^{t+1}, \mathbf{y}^{t+1})}{D^G(\mathbf{x}^t, \mathbf{y}^t)} \\
&= GM^{t,t+1}(\mathbf{x}^t, \mathbf{y}^t, \mathbf{x}^{t+1}, \mathbf{y}^{t+1}).
\end{aligned} \tag{14}$$

The relationship between the ML and the M indices is trivial. □

References

Asmild M, Tam F (2007) Estimating global frontier shifts and global Malmquist indices. J Prod Anal 27:137–148

Balk B (1998) Industrial price, quantity, and productivity indices: the micro-economic theory and an application. Kluwer, Boston.

Battese GE, Rao DSP, O'Donnell CJ (2004) A metafrontier production function for estimation of technical efficiencies and technology gaps for firms operating under different technologies. J Prod Anal 21:91–103

Berg SA, Førsund FR, Jansen ES (1992) Malmquist indices of productivity growth during the deregulation of norwegian banking, 1980–1989. Scand J Econ 94:S211–S228

Chen Y (2005) Measuring super-efficiency in DEA in the presence of infeasibility. Eur J Oper Res 161:545–551

Chung YH, Färe R, Grosskopf S (1997) Productivity and undesirable outputs: a directional distance function approach. J Environ Manage 51:229–240

Fan Y, Ullah A (1999) On goodness-of-fit tests for weakly dependent processes using kernel method. J Nonparametr Stat 11:337–360

Färe R, Grosskopf S, Norris M, Zhang Z (1994) Productivity growth, technical progress, and efficiency change in industrialized countries. Am Econ Rev 84:66–83

Färe R, Grosskopf S, Pasurka CA Jr (2001) Accounting for air pollution emissions in measures of state manufacturing productivity growth. J Reg Sci 41:381–409

Färe R, Grosskopf S, Pasurka CA Jr (2007) Environmental production functions and environmental directional distance functions. Energy 32:1055–1066

IPCC (2007) Climate change 2007: the physical science basis. Cambridge University Press, New York

Jeon BM, Sickles RC (2004) The role of environmental factors in growth accounting. J Appl Econ 19:567–591

Kumar S (2006) Environmentally sensitive productivity growth: a global analysis using Malmquist-Luenberger index. Ecol Econ 56:280–293

Kuosmanen T (2005) Weak disposability in nonparametric production analysis with undesirable outputs. Am J Agric Econ 87:1077–1082

Kuosmanen T, Podinovski V (2009) Weak disposability in nonparametric production analysis: reply to Färe and Grosskopf. Am J Agric Econ 91:539–545

Lall P, Featherstone AM, Norman DW (2002) Productivity growth in the Western Hemisphere (1978–1994): the Caribbean in perspective. J Prod Anal 17:213–231

Luenberger DG (1992) Benefit functions and duality. J Math Econ 21:461–481

Nakano M, Managi S (2008) Regulatory reforms and productivity: an empirical analysis of the Japanese electricity industry. Energy Policy 36:201–209

O'Donnell C, Rao DSP, Battese G (2008) Metafrontier frameworks for the study of firm-level efficiencies and technology ratios. Empir Econ 34:231–255

Oh DH, Lee JD (2010) A metafrontier approach for measuring Malmquist productivity index. Empir Econ 32:46–64

Pastor J, Lovell C (2007) Circularity of the Malmquist productivity index. Econ Theory 33:591–599

Pastor JT, Lovell CAK (2005) A global Malmquist productivity index. Econ Lett 88:266–271

Tulkens H, Vanden Eeckaut P (1995) Non-parametric efficiency, progress and regress measures for panel data: methodological aspects. Eur J Oper Res 80:474–499

Weber W, Domazlicky B (2001) Productivity growth and pollution in state manufacturing. Rev Econ Stat 83:195–199

Xue M, Harker PT (2002) Note: ranking dmus with infeasible super-efficiency DEA models. Manage Sci 48:705–710

Yörük BK, Zaim O (2005) Productivity growth in OECD countries: a comparison with Malmquist indices. J Compar Econ 33:401–420

Yu MM, Hsu SH, Chang CC, Lee DH (2008) Productivity growth of Taiwan's major domestic airports in the presence of aircraft noise. Logist Transp Rev 44:543–554

Zhou P, Ang BW, Han JY (2010) Total factor carbon emission performance: a Malmquist index analysis. Energy Econ 32:194–201